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# Fuzzy Surrogate Safety Metrics for real-time assessment of rear-end collision risk. A study based on empirical observations

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ABSTRACT

The present paper discusses two fuzzy Surrogate Safety Metrics (SSMs) for rear-end collision, the Proactive Fuzzy SSM (PFS) and Critical Fuzzy SSM (CFS). The objective is to investigate their applicability for evaluating the real-time rear-end risk of collision of vehicles to support the operations of advanced driver assistance and automated vehicle functionalities (from driving assistance systems to fully automated vehicles). The proposed Fuzzy SSMs are evaluated and compared to other traditional metrics on the basis of empirical observations. To achieve this goal, an experimental campaign was organized in the AstaZero proving ground in Sweden. The campaign consisted of two main parts: a car-following experiment with five vehicles solely driven by Adaptive Cruise Control (ACC) systems and a safety critical experiment, testing the response of the Autonomous Emergency Braking (AEB) system to avoid collisions on a static target. The proposed PFS is compared with the safe distance defined by the well-known Responsibility Sensitive Safety (RSS) model, showing that it can produce meaningful results in assessing safety conditions also without the use of crisp safety thresholds (like in the case of RSS). The CFS outperformed the well-known Time-To-Collision (TTC) SSM in the a-priori identification of the cases, where the tested vehicles were not able to avoid the collision with the static target. Moreover, results show that CFS at the time of the first deceleration is correlated with the velocity of the vehicle at the time of collisions with the target.

## 1. Introduction

It is estimated that more than 90 % of road accidents are due to human error (Critical Reasons for Crashes Investigated in the National Motor Vehicle Crash Causation Survey, 2018). The driving ability of human drivers can be compromised by fatigue, distraction and other causes (Fountas et al., 2019). For this reason, in the last decades, there has been strong political will to support the deployment of Advanced Driving Assistance Systems (ADAS) and Automated Vehicle (AV) functionalities, taking partial or complete control of the driving operation in specific situations (SAE International, 2017). Vehicle automation and connectivity is expected to significantly contribute to traffic safety, network efficiency, environmental sustainability and equity in access to transportation for road users (The Future of Road Transport, 2019). Without waiting for full automation, ADAS such as the Forward Collision Warning (FCW) and the Autonomous Emergency Braking (AEB) systems can reduce the risk of collision of up to 50 % (Cicchino, 2017). However, in reality, this assumption does not take into account the unknown number of accidents avoided due to the proactive perception of human drivers and their ability to successfully deal with challenging driving situations. Consequently, the safety impact of introducing increasing levels of vehicles' automation is still very uncertain. It is expected that automated controllers will certainly need time to be at least as safe as human drivers (Shladover, 2019).

Prior to mass deployment, highly automated vehicles need to demonstrate their capability to drive safely and regulators are working to put in place the right tools for being able to verify this aspect. Characterizing the safety level of an ADAS or AV controller, however, is not a trivial task. In addition, automated driving behavior should not be bounded to very conservative rules, as the traffic conditions can

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significantly deteriorate (Mattas et al., 2019a) and the AV acceptance in the case of reduced traffic efficiency would be jeopardized. Furthermore, safety should not just be demonstrated at the end of the development phase and prior to the introduction of the vehicle to the market but should hold for the entire vehicle lifetime. Given the expected frequent updates that the software governing vehicles' operations will receive, a robust, as well as, flexible solution able to guarantee vehicle safety in all situations within its operational design domain (ODD) is required.

A possible approach recently proposed is to define a set of analytical rules describing a safety envelope that a vehicle is requested to respect at any time in order not to be responsible for causing an accident. Different analytical frameworks for the vehicle safety envelope have been recently proposed by the industry (Shalev-Shwartz et al., 2017; Safety Force Field for Autonomous Vehicles, 2019). In addition, a number of different surrogate safety metrics (SSMs) exist in the literature (Laureshyn et al., 2010) and could also be useful to define a safety envelope. SSMs have been used since the late' 60 (Hyden, 1987; Hayward, 1971), to count the number of conflicts and predict the number of road accidents using either real (for traffic monitoring) or simulated (for impact assessment studies) vehicle trajectories. It has been shown that the number of conflicts is statistically correlated to the number of accidents (Tarko, 2018). This is an important asset of SSMs, as accident data is often of poor quality (Mannering and Bhat, 2014). Moreover, SSMs are also used by vehicle controllers of AVs or ADAS to identify safety critical situations and react accordingly (Milanés et al., 2012). For an extensive review please refer to (Mahmud et al., 2017; Mullakkal-Babu et al., 2017). It must be noted that traditional traffic conflict techniques have been developed to overcome the lack of good quality accident data and have relied on human observers (Laureshyn et al., 2010). Hence, many parameters that are not always available are often omitted. One of the most significant is the reaction time that is not taken into account by many of the existing surrogate safety metrics, and has been shown to be critical when it comes to safety (Li et al., 2017; Kuang et al., 2015). Assuming that the response time of an automated controller can be much faster and more consistent than that of a human driver, this is a significant limitation. Moreover, most SSMs represent the proximity to an accident and not the severity of the accident and the safe and unsafe situations are not clearly distinguished (Laureshyn et al., 2017). The classification is usually done with crisp thresholds, that are often arbitrary. For example, in the case of TTC the thresholds values used in the literature vary in the range [1.5,4] sec (Mahmud et al., 2018). This, and other shortcomings of traditional SSMs have been mentioned in the literature. For example, Kang et al. (Kuang et al., 2015) already discussed the arbitrary thresholds, boundaries regarding relative speed and ignoring the reaction time. They have proposed a surrogate measure named Aggregated Crash Index (ACI) based on a tree structure that describes 8 different conflict types. They have validated their model based on data from a freeway section of Pacific Motorway, Australia.

Those disadvantages may not be critical when the objective is to statistically estimate the number of accidents in a road section. Indeed, analytical relationships correlating the number of conflicts with the number of accidents on a specific road section have been statistically defined. However their applicability to new road designs or significant infrastructural modifications is still limited (Tarko, 2019). Therefore attempts have been made to define models that are not site specific (Zheng et al., 2019). However, when traffic conflict techniques are used to define a safety envelope around a vehicle to classify its safety in real-time, these limitations are no longer acceptable, as a potential error can be extremely dangerous.

The scope of the paper is to present a comprehensive analytical framework for the assessment of the safety of ADASs and AVs for rear-end collisions (Mattas et al., 2018a), based on fuzzy logic, and to compare its performance with one of the recently proposed framework for safety envelope and a well-known and widely used surrogate safety metric using empirical observations.

Fuzzy logic was introduced in the work of Zadeh (Zadeh (1965)), and has found applications in many fields, including accident modeling (Dimitriou and Vlahogianni, 2015). Based on this framework, two fuzzy SSMs have been developed, the Proactive Fuzzy Surrogate Safety Metric (PFS) and the Critical Fuzzy Surrogate Safety Metric (CFS) (Mattas et al., 2019b). Fuzzy logic can bring significant added value for this type of analysis as it offers simple and flexible tools to cope with uncertainty. Counter intuitive crisp thresholds classifying situations to be certainly safe or unsafe are avoided. Different levels of safety are introduced, that can be evaluated and numerically handled using fuzzy arithmetic.

The empirical observations have been collected from an ad-hoc experimental campaign organized in the AstaZero proving ground in Sweden (AstaZero, 2020). The campaign consists of two main parts: i) a car-following test involving five vehicles driven by Adaptive Cruise Control (ACC) systems (Makridis et al., 2020) and ii) a series of safety critical tests where the same vehicles use the Autonomous Emergency Braking (AEB) system to avoid collision with a static target. Additional tests were conducted to measure the hardest possible deceleration capabilities of the vehicles used. The first part was post-processed to create a synthetic database of events that can potentially become critical if the front vehicle performs a hard deceleration and it was used to assess the proactive metrics. The second part was used to directly assess the critical metrics. On the same empirical observations, apart from the proposed metrics, the authors implemented and tested the Responsibility Sensitive Safety Model (RSS) developed by Intel/Mobileye (Mobileye (2018)) on the first part, and the time to collision (TTC) metric, that is one of the most widely used critical SSMs in the literature (Mahmud et al., 2018; Laureshyn et al., 2016), on the second part. The SSMs are assessed based on individual vehicle behavior in experiments with a few vehicles. Therefore, in this stage no correlation can be made between conflicts and accident frequency. Such a study would not be trivial, considering the small number of vehicles with automated functionalities in the traffic networks today.

The paper is organized as follows: in the next section the Fuzzy Surrogate Safety Metrics developed are presented. In section 3 time to collision (TTC) and Responsibility Sensitive Safety Model (RSS), the metrics to which the proposed fuzzy metrics will be compared are presented. The experimental campaign is described in section 4. The methodology used to compare the metrics on the empirical data is detailed in section 5. The results are presented in section 6. Finally, Section 7 presents our conclusion

## 2. Fuzzy Surrogate Safety Metrics

For two vehicles traveling in the same direction, the Fuzzy Surrogate Safety Metrics described in the present paper consider two critical distances, the maximum unsafe distance and the minimum safe distance, that will be defined later in the paper. Then, the fuzzy set "unsafe situation" is created as in Eq. (1).

$$\mu_A(d) = \begin{cases} 1\ ,\ 0 \le d < d_{unsafe} \\ 0\ ,\ d \ge d_{safe} \\ \dfrac{d - d_{safe}}{d_{unsafe} - d_{safe}},\ d_{unsafe} < d < d_{safe} \end{cases} \tag{1}$$

where $d$ is the inter-vehicle distance, $d_{unsafe}$ and $d_{safe}$ the maximum unsafe distance and the minimum safe distance, $\mu_A$ the membership function. Thus, a situation can be identified as certainly safe when the two vehicles are far enough or certainly unsafe when they are too close. Additionally, there is a margin where the vehicles can be unsafe with a membership value. For situations that are not entirely safe, a fuzzy interval can be produced, representing the "unsafe distance". The interval can be of type B or C as in Eqs. (2) and (3). Type B intervals are trapezoidal with their maximum membership value equal to 1, while Type C, are triangular with their maximum membership value less than 1. Both are presented in Fig. 1. Different types of fuzzy intervals could be used as gaussian or sigmoid. The choice to use those specific shapes was made to facilitate further calculations.

$$\mu_B(x) = \begin{cases} 1\,, & 0 \le x < d_{unsafe} - d \\ 0\,, & x \ge d_{safe} - d \\ \dfrac{x - d_{safe} - d}{d_{unsafe} - d_{safe}}, & d_{unsafe} - d < x < d_{safe} - d \end{cases} \tag{2}$$

$$\mu_C(x) = \begin{cases} 0\,, & x \ge d_{safe} - d \\ \dfrac{x - d_{safe} - d}{d_{unsafe} - d_{safe}}, & 0 \le x < d_{safe} - d \end{cases} \tag{3}$$

The fuzzy intervals of type $\mu_C$ are subnormal, as the membership function never reaches the value 1. For this reason, new fuzzy intervals denoted $\mu_{C'}$ are created as shown in Eq. 4, as the strong union of any of $\mu_c$ and a fuzzy singleton $s$ where $s(x)$ is 1 for $x = 0$ and zero when not. Those fuzzy intervals have the required attributes to be used as a fuzzy SSM.

$$\mu_{C'}(x) = \begin{cases} 1\,, & x = 0 \\ 0\,, & x \ge d_{safe} - d \\ \dfrac{x - d_{safe} - d}{d_{unsafe} - d_{safe}}, & 0 < x < d_{safe} - d \end{cases} \tag{4}$$

The produced fuzzy intervals have a number of attributes that are important for their function as surrogate safety metrics:

- **Support**, namely he interval for which the membership function takes a non-zero value. The support of such fuzzy intervals is always finite, and represents the difference between the minimum safe distance and the actual inter-vehicle distance, hence can be used as a proxy of the severity of the situation.
- **Core**, namely the interval for which the membership function takes the value 1. If the core of a fuzzy SSM is single value 0, there was no instance where the distance was equal or smaller to the maximum unsafe distance. For dangerous situations, the diameter of the core of a FSSM can be used as another proxy of the severity of the situation.
- **α-cuts**: Any convex fuzzy set $X$ can be defined by the α-cuts, $X^\alpha$. For every real number $\alpha \in [0,1]$, there is a crisp interval $X^\alpha(\alpha)$, for which $\mu(x) \ge \alpha$. The α-cuts can be interpreted as intervals of confidence (Hanss, 2020). Furthermore, the α-cuts are useful for fuzzy intervals addition using interval arithmetic (Moore and Lodwick, 2003).

The Fuzzy SSMs developed this way are countably additive. Countable additivity is one of the criteria that a function should satisfy to be considered a measure (Measure Theory and Fine Properties of Functions, 2019). This makes the comparison between large scenarios, composed by a big number of safe and unsafe instances, more straightforward.

Two distances need to be defined, the maximum unsafe and minimum safe distance, bringing up the question “What is a safe following distance?” . The Vienna Convention on Road Traffic defines a ‘safe

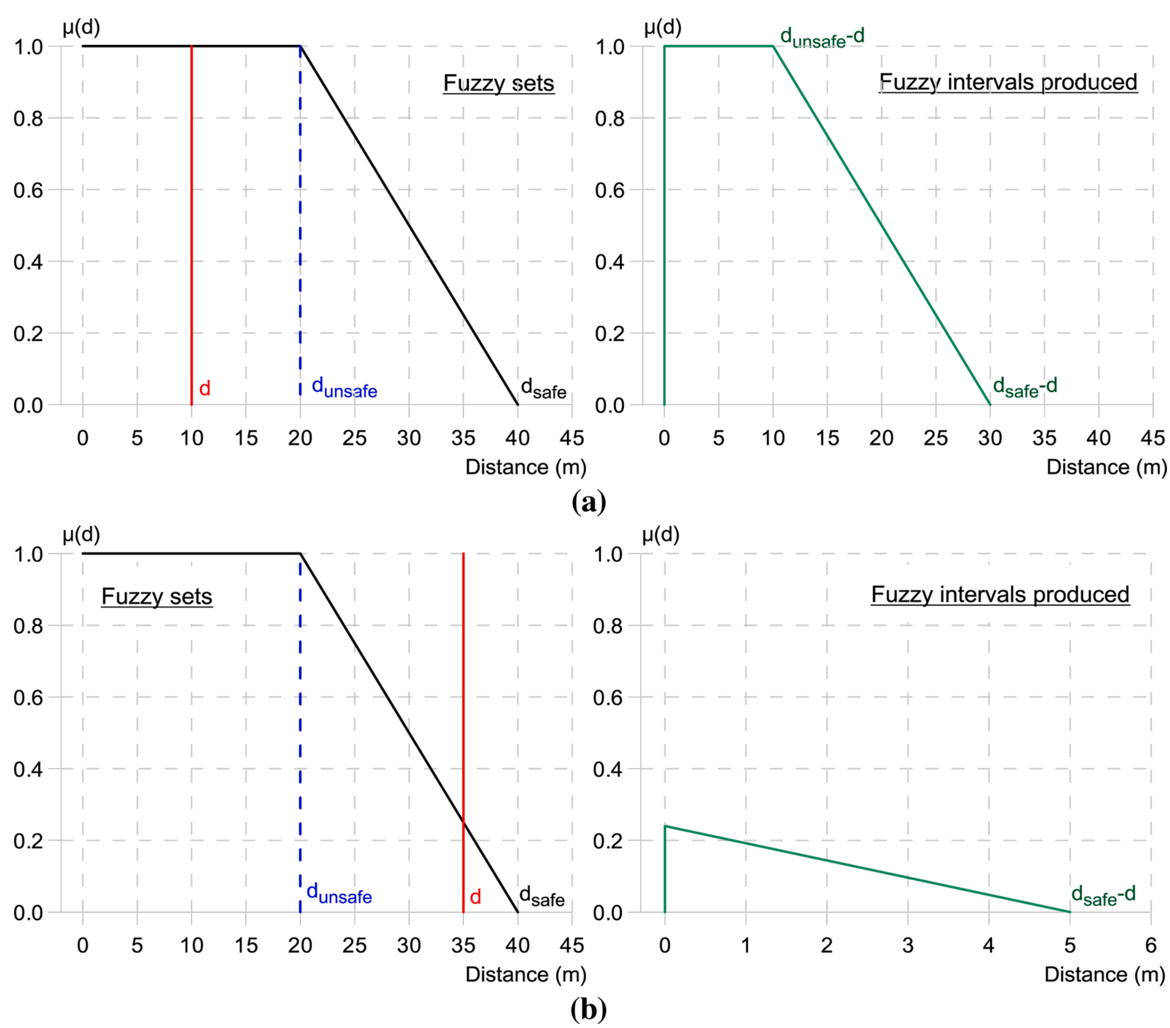


**Fig. 1.** “Unsafe situation” fuzzy set and produced fuzzy intervals (a) type B, (b) type C.

distance' as the distance such that *"a collision between vehicles can be avoided if the vehicle in front performs an emergency brake"* (Vanholme et al., 2013). However, there is also the following definition of traffic conflicts according to the Association for International Cooperation on Traffic Conflicts Techniques (Güttinger, 1984). *"A traffic conflict is an observable situation in which two or more road users approach each other in space and time to such an extent that there is a risk of collision if their movements remain unchanged."* From the first definition, a situation is unsafe even if there is no imminent danger of accident, so it can be characterized as proactive safety. Meanwhile, according to the second definition a situation is unsafe if it is already critical and action must be taken to avoid the accident. Hence, SSMs can be categorized in proactive and critical safety metrics according to their formulation. When it comes to the safety analysis of different driving automation systems, or to their design, both critical and proactive surrogate safety metrics can be important.

Therefore, two metrics have been developed, a Proactive Fuzzy Surrogate Safety Metric (PFS) and a Critical Fuzzy Surrogate Safety Metric (CFS). It is worth noticing that the two metrics concern rear-end collisions of two vehicles in car-following conditions. Therefore, the safety level of other traffic situations (e.g. vehicles cutting in from an adjacent lane, side-impacts or vehicles travelling in opposite directions) cannot be evaluated

### 2.1. Proactive Fuzzy Surrogate Safety Metric

Let two vehicles $c_f$ and $c_r$ traveling on the same lane with $c_f$ being the front vehicle and $c_r$ the rear in a car following situation. $c_r$ is obliged to keep enough distance so that if $c_f$ decelerates, $c_r$ he has enough time to perceive and react. This behavior is measured by the PFS. The formulation of the maximum unsafe distance and the minimum safe distance are reported in **Equations 5** and **6**:

$$d_{safe}(t) = u_r(t)\tau + \frac{u_r^2(t)}{2b_{r,comf}} - \frac{u_f^2(t)}{2b_{f,max}} \tag{5}$$

$$d_{unsafe}(t) = u_r(t)\tau + \frac{u_r^2(t)}{2b_{r,max}} - \frac{u_f^2(t)}{2b_{f,max}} \tag{6}$$

with $u_r$ being the speed of the rear vehicle, $u_f$ the speed of the front vehicle, $\tau$ the reaction time of the rear vehicle, $b_{r,comf}$ the comfortable deceleration of the rear vehicle, $b_{r,max}$ the maximum deceleration of the rear vehicle and $b_{f,max}$ the maximum deceleration of the leading vehicle. The front vehicle's maximum deceleration must be considered at least as hard as the maximum deceleration of the rear vehicle. The assumption is that when the vehicle $c_f$ starts decelerating with the maximum possible deceleration, the vehicle $c_r$ continues driving with constant speed for time $\tau$ and then starts to decelerate with its comfortable deceleration. Both vehicles decelerate until they come to a stop, as this is the worst-case scenario. If the distance is enough for $c_r$ to stop and avoid the collision, then the distance is 'certainly' safe. On the other hand, if vehicle $c_r$ after reaction time decelerates as hard as possible and still does not avoid an impact, the distance is 'certainly' unsafe.

### 2.2. Critical Fuzzy Surrogate Safety Metric

According to the second definition of an unsafe situation, the CFS identifies situations where a collision is imminent and action is necessary to avoid an accident. The formulation of the maximum unsafe distance and the minimum safety distance are presented in **Equations 7** and **12**:

$$a_r^{'}(t) = \max\left(a_r(t),\ -b_{r,comf}\right) \tag{7}$$

$$u_r(t+\tau) = u_r\ a_r^{'}(t) \tag{8}$$

If $u_r(t+\tau) \le u_f(t)$:

$$d_{safe}(t) = d_{unsafe}(t) = \frac{\left(u_r(t) - u_f(t)\right)^2}{2a_r^{'}(t)} \tag{9}$$

Else if $u_r(t+\tau) > u_f(t)$:

$$d_{new} = \left(\frac{(u_r(t) + u_r(t+\tau))}{2} - u_f(t)\right)\tau \tag{10}$$

$$d_{safe}(t) = d_{new} + \frac{\left(u_r(t) + a_r^{'}(t)\tau - u_f(t)\right)^2}{2b_{r,comf}} \tag{11}$$

$$d_{unsafe}(t) = d_{new} + \frac{\left(u_r(t) + a_r^{'}(t)\tau - u_f(t)\right)^2}{2b_{r,max}} \tag{12}$$

with $u_r$ the speed of the rear vehicle, $u_f$ the speed of the front vehicle, $\tau$ the reaction time of the rear vehicle, $b_{r,comf}$ the comfortable deceleration of the rear vehicle, $b_{r,max}$ the maximum deceleration of the rear vehicle and $b_{f,max}$ the maximum deceleration of the leading vehicle. The front vehicle's maximum deceleration must be at least as hard as the maximum deceleration of the rear vehicle or harder. The assumption is that if the vehicle $c_f$ keeps constant speed, the vehicle $c_r$ continues driving with constant acceleration for time $\tau$, and then starts to decelerate with its comfortable deceleration. If the distance is enough for $c_r$ to stop before crashing, the distance is certainly safe. On the other hand, if the vehicle $c_r$ after reaction time decelerates as hard as possible and still does not avoid an impact, the distance is certainly unsafe. The constant acceleration assumption for the follower during the reaction time separates the cases of the follower already accelerating or decelerating, with the former being more dangerous than the latter.

## 3. Benchmark Safety Metrics

The performance of the two FSSMs has been assessed by comparing them with two well know safety metrics on empirical data. In particular, the PFS metric was compared to the safe distance requirement of the Responsibility Sensitive Safety Model (RSS), while the CFS metric has been compared with the well-known Time to Collision (TTC). In the next section, these tow metrics are briefly presented for the reader's convenience.

### 3.1. Time to Collision (TTC)

TTC identifies possible conflicts by calculating the time it would take for two vehicles to collide if they would continue their movement with same constant speed as shown in **Equation 13** (Driver Metrics, 2020):

$$TTC = \frac{d}{\Delta u} \tag{13}$$

with $d$ being the distance between the two vehicles and $\Delta u$ their relative speed. The calculation can only happen when the rear vehicle is faster than the front vehicle. On the basis of the TTC value, a situation is considered unsafe if TTC is lower than a crisp threshold quantified from the literature in the range [1.5,4] sec (Mahmud et al., 2018).

### 3.2. Responsibility Sensitive Safety Model (RSS)

RSS provides the mathematical formulation of the duty of care. In other words, an AV abiding to the RSS rules could be involved in an accident, but according to the formulation, cannot be responsible for it.

The formulation of the safe longitudinal distance of RSS can be used as a proactive safety metric, classifying safe and potentially unsafe situations. Let two vehicles $c_r$ and $c_f$ traveling in the same direction, one before the other with $c_f$ being in front. The inter-vehicle distance can be considered as safe only if assuming that $c_f$ decelerates abruptly to come to a stop, $c_r$ has enough space to avoid the collision. Since $c_r$ cannot react before a time equal to its reaction time, the worst case scenario is that it has been accelerating with its maximum acceleration. So, the minimum safe distance is formulated in **Equation 14** (Shalev-Shwartz et al., 2017):

$$d_{min} = \left[u_r\tau + \frac{1}{2}a_{max,accel}\tau^2 + \frac{\left(u_r + \tau a_{max,accel}\right)^2}{2a_{min,brake}} - \frac{u_f^2}{2a_{max,brake}}\right] \tag{14}$$

with $u_r$ being the speed of the rear vehicle, $u_f$ the speed of the front vehicle, $\tau$ the reaction time of the rear vehicle, $a_{max,accel}$ the maximum acceleration of the rear vehicle, $a_{min,brake}$ the deceleration of the rear vehicle and $a_{max,brake}$ the maximum deceleration of the leading vehicle. If the rear vehicle is inside the unsafe distance it must decelerate at least with $a_{min,brake}$.

In a more recent publication based on the RSS, the Automatic Preventive Braking (APB) system was proposed (Shalev-Shwartz et al., 2018). The APB should be an ADAS that would intervene when a vehicle driven by a human keeps unsafe distance to the front vehicle. The APB would decelerate the vehicle with a constant jerk until it is stopped or out of the unsafe distance. The deceleration should be milder than that of an AEB, so the system can operate for higher speeds without requiring frequent emergency braking maneuvers. Moreover, the safe distance formula is changed as the stopping distance takes into account also a jerk limit. The new stopping distance of the rear vehicle is described on Eqs. 15 and 16 (Shalev-Shwartz et al., 2018):

$$d_{stop} = \left[u_0 T + \frac{1}{2}a_0 T^2 - \frac{1}{6}j_{max}T^3\right] + \frac{\left(u_0 + a_0 T - \frac{1}{2}j_{max}T^2\right)^2}{2a_{min,brake}} \tag{15}$$

$$T = \min\left(\frac{a_0 + a_{min,brake}}{j_{max}}, \frac{a_0 + \sqrt{a_0^2 + 2j_{max}T^2}}{j_{max}}\right) \tag{16}$$

with $d_{stop}$ being the stopping distance, $u_0$ and $a_0$ the velocity and acceleration of the rear vehicle, $j_{max}$ the maximum absolute jerk, $a_{min,brake}$ the deceleration of the rear vehicle and T the time it takes for the vehicle either to reach the $a_{min,brake}$ or to stop.

## 4. Experimental campaign

The experimental campaign has been carried out in the AstaZero proving ground in Sweden (AstaZero, 2020). The campaign consisted of two main parts: a car-platoon of five vehicles driven by Adaptive Cruise Control (ACC) systems and a number of safety related critical tests where the same vehicles use the Autonomous Emergency Braking (AEB) system to avoid collision with a static target. Additional tests were conducted to measure the hardest possible deceleration of each vehicle imposed by a human driver. The data acquisition system used was the RT-Range S multiple target ADAS measurements solution by OXTS (DGPS Archives, 2019) with speed horizontal accuracy around 0.01 m/s and horizontal accuracy for the vehicles' position around 0.02 m.

### *4.1. Adaptive cruise control platooning*

Five vehicles were driving in a car-platoon formation around the rural road in AstaZero for 27 laps. All vehicles were driven by ACC and the leading vehicle kept constant desired speed of 70 km/h for 6 laps, while creating small perturbations accelerating and decelerating using only the ACC settings for the rest of the 21 laps. The perturbations were performed between 50 km/h and 100 km/h. More information on the platooning experiment, the ACC behavior, reaction time and instability of the fleet can be found in (Makridis et al., 2020).

### *4.2. Emergency braking*

The test of the AEB involved a straight line of road were each of the five vehicles was manually driven to approach a soft target with constant speed, until the AEB applied braking in the attempt to avoid colliding with the target. Every vehicle has been tested by approaching the target with constant speed of 20, 42, 50 and 60 km/h. These tests have been repeated three times. In the first case the target vehicle was perfectly aligned with the trajectory of the vehicle under test. In the second and third cases the position of the target vehicle had an offset of 1 m with respect to the position of the vehicle under test, to the left or to the right. The resulted emergency braking database consisted of 49 experiments (not all the combinations were performed for the five vehicles under test). In 10 out of 49 cases, the vehicle failed to stop before reaching contact with the static target, leading to a collision. In one of these cases the safety driver, when the imminent collision was unavoidable, had to execute an emergency evasion maneuver to avoid serious damages to the vehicle.

### *4.3. Vehicles' braking capability*

The same vehicles were tested for their manual braking capabilities by accelerating from 0 km/h to 40 km/h, 80 km/h and 120 km/h and then decelerating manually as hard as possible, to model a driver performing an emergency braking.

The braking capability of the vehicles was measured to be used in the validation process and it was found to be similar for all five vehicles. This can be seen in Fig. 2, which shows the relationship between the stopping distance and the velocity of each vehicle at the beginning of the deceleration for both AEB and manual braking. Moreover, it was observed that the vehicles achieve stronger deceleration rates for manual braking than for emergency braking. In the first case, the vehicles reached on average 12 $m/s^2$ maximum deceleration and average jerk value of approximately 30 $m/s^3$, while for AEB, the corresponding values of deceleration and jerk were 9 $m/s^2$ and 20 $m/s^3$ respectively. It is worth noting that the experiment was carried out under ideal conditions, inside a test track with perfect road surface and involving well-maintained high-end vehicles. The maximum deceleration was on average 12 $m/s^2$ and the maximum jerk 30 $m/s^2$. For real-world conditions, further investigation might be needed to adjust the above-mentioned values accordingly.

Furthermore, in Fig. 2a) and b), two physical models of estimating the stopping distance based on the initial speed are shown against the data. On the green line, the stopping distance is calculated only using the maximum deceleration value, assuming infinite jerk. This assumption is made by many microsimulation models (Gipps, 1981; Kesting et al., 2010), SSMs like the DSS (Japan Society of Traffic Engineers, 2005), the RSS safe distance shown in Eq. 14, and in the safe and unsafe distance for both proposed Fuzzy SSMs. This assumption of no minimum jerk boundary results in underestimating the stopping distance because in reality the deceleration at the first instances is softer and takes a short but not negligible time to reach its maximum value. The second physical model, represented by the blue line takes into account also the maximum negative jerk capability, similarly to the RSS for APB formulation in Eq. 15 and fits the observed values much better. It has to be noted that AEB in most cases works for speeds up to 20 m/s, so there were no data for higher speeds. When AEB will be available for higher speeds the behavior may be different to what is presented in Fig. 2 a), where it is only assumed that the maximum deceleration and jerk would not change.

## 5. Benchmarking methodology

In the present section, the methodology adopted to compare the two sets of safety metrics is described.

### *5.1. Synthetic collision courses*

The value of PFS shows the potential of an impact and its severity if

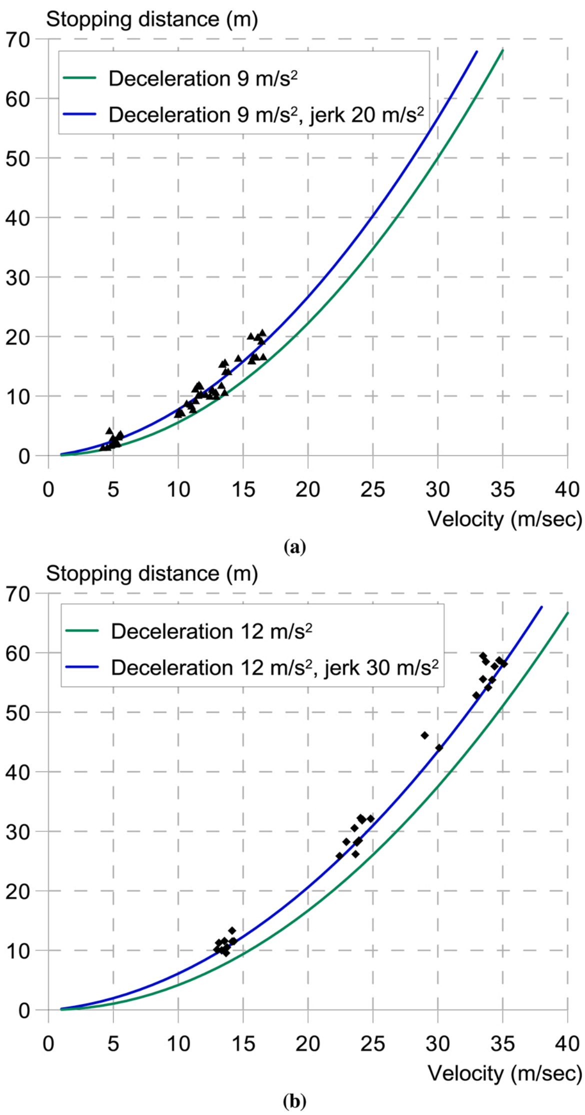


**Fig. 2.** Stopping distance relative to initial speed for a) AEB, b) manual braking.

the leader vehicle decelerates sharply. To test this against real data, a leader vehicle would have to perform an emergency braking in front of a following ACC vehicle, on many different scenarios. However, this could lead to destructive tests, bringing the vehicles to actual collision. This was not possible or available in the described experimental campaign. Therefore, synthetic emergency braking decelerations are simulated based on the ACC platoon data and the hard decelerations observed, to recognize potentially unsafe situations.

Each vehicle pair of leader and follower is investigated separately. Hence, the experiment includes 4 pairs of leader and follower vehicles according to the experimental description provided above. The stopping distance models using the maximum deceleration and average jerk were used to simulate possible severe decelerations from the front vehicle of each pair. The leader vehicle's stopping distance is modelled according to the hardest manual braking. Then the corresponding reaction from the rear vehicle was modelled assuming the AEB activation. Finally, the distance travelled by the rear vehicle for the initial 0.2 s is added to take into account an assumed reaction time of the vehicle.

This process was done for every instance of the platooning data, and the cases in which the vehicles would collide according to the simulated braking are considered unsafe. This is just an estimation of the actual safety, applied in post process and using detailed data on the braking capabilities of each vehicle. It cannot be applied real time, but it can provide a benchmark for the comparison of the safety metrics under assessment. Out of 257,384 cases, 1,510 resulted to be unsafe, as the distance was not enough for the rear vehicle to react. The same calculations have been carried out for the rear vehicle decelerating with maximum deceleration of 3 m/s$^2$ and 6 m/s$^2$ to identify how hard a deceleration is required at any instance from the rear vehicle in the case of an emergency braking by the front.

### 5.2. Model parameters

For both RSS and PFS safety critical distances, the reaction time of the rear vehicle is an important input. For the ACC platooning, the reaction time of the controller can be estimated by correlating a shifted series of the input for the controller (namely the relative velocity and distance of the vehicles), to the output of the controller (namely the acceleration). In the literature, the reaction time of commercially-available ACC systems is estimated to be close to 1 s or even higher (Makridis et al., 2018). However, to the best of the authors' understanding, this is a decision of the manufacturers seeking a comfortable travelling experience for the passengers, and not a constraint of the software or mechanics of the system. Hence, for safety critical situations, especially considering future automated driving systems, the controller's reaction can be faster, so the reaction time value used in the present paper has been set to 0.2 s.

Another input required for both RSS and PFS calculation is the vehicle deceleration. $a_{max,brake}$ (for RSS) and $b_{f,max}$ (for PFS), both represent the maximum deceleration capability of the vehicle in front. This is assumed to be equal to the aforementioned average deceleration value of the manual braking observed, which is 12 m/s$^2$. For PFS, $b_{r,comf}$ and $b_{r,max}$ represent the maximum comfortable deceleration and the maximum possible deceleration that the vehicle can achieve. ACC controlled vehicles have been observed not to use decelerations higher than 3 m/s$^2$, which is also a value used in the existing literature (Mattas et al., 2018b). For this reason this value is used as $b_{r,comf}$ as it can be argued that it is a good estimation of a boundary for comfortable deceleration. The $b_{r,max}$ is assumed to be equal to the average deceleration value of the AEB observed, which is 9 m/s$^2$.

For the RSS, only one deceleration value is used for the rear vehicle, the $a_{min,brake}$, and its value is not specified by the developers of the model (Shalev-Shwartz et al., 2017). It is both the deceleration used for the evaluation of the safe distance, and the deceleration applied to the rear vehicle if the inter-vehicle distance becomes unsafe. Hence, it can be argued that a realistic value of maximum deceleration to be used to assess safety, as for $b_{r,max}$, can be 9 m/s$^2$. However, taking into account that RSS and the derivative APB system are according to the authors designed to be proactive and not to force very hard decelerations that would maybe create unsafe situations to the vehicles upstream, a more comfortable deceleration of 3 m/s$^2$ as per the $b_{r,comf}$ was also used and two versions of the RSS model, RSS1 and RSS 2 were tested, using $a_{min,brake}$ equal to 9 m/s$^2$ and 3 m/s$^2$ respectively. Two more models are used, APB 1 and APB 2, based on the more recent definition of RSS safe distance that takes into account the jerk (Shalev-Shwartz et al., 2018). The jerk value is estimated to be 20 m/s$^3$ as observed from the emergency braking experiments and the $a_{min,brake}$ equal to 9 m/s$^2$ and 3 m/s$^2$ as for RSS 1 and RSS 2. All the parameter values used are presented in

**Table 1**
SSM parameters for proactive safety.

| Model | $\tau$ | $a_{min,brake}$ | $a_{max,brake}$ | jerk | $b_{r,comf}$ | $b_{r,max}$ | $b_{f,max}$ |
|---|---|---|---|---|---|---|---|
| PFS | 0.2 | – | – | – | 3 m/s$^2$ | 9 m/s$^2$ | 12 m/s$^2$ |
| RSS 1 | 0.2 | 9 m/s$^2$ | 12 m/s$^2$ | – | – | – | – |
| RSS 2 | 0.2 | 3 m/s$^2$ | 12 m/s$^2$ | – | – | – | – |
| APB 1 | 0.2 | 9 m/s$^2$ | 12 m/s$^2$ | 20 m/s$^3$ | – | – | – |
| APB 2 | 0.2 | 3 m/s$^2$ | 12 m/s$^2$ | 20 m/s$^3$ | – | – | – |

Table 1.

For the calculation of CFS, the reaction time of the controller is assumed to be 0.2 s and the decelerations $b_{r,comf}$ and $b_{r,max}$ 3 m/s$^2$ and 9 m/s$^2$respectively. The assumptions are the same as in the case of the PFS. For the calculation of TTC there are no parameter values to be assumed.

### 5.3. Examining the suitability of SSMs

#### 5.3.1. Proactive fuzzy surrogate safety metric

For the Proactive SSMs, there is a need to correctly identify situations that are potentially dangerous. Using the synthetic emergency breaking, PFS and all the RSS derivative formulas have been used to classify the situations into safe and unsafe. The accuracy and precision of the classification are used to evaluate the different models. For the PFS metric, for the sake of comparison, every case that is unsafe with membership value larger or equal to 0.95 is considered unsafe and all others are safe, creating a threshold value and denoted as PFS 0.95. Afterwards, the complete spectrum of PFS membership value was shown against the result of the synthetic collision courses with different deceleration values, to present the advantage of using a fuzzy SSM.

#### 5.3.2. Critical fuzzy surrogate safety metric

The CFS identifies conditions where the collision is imminent. As such it will be examined using the trajectories of the experiments where vehicles used AEB in front of a static target and will be compared to the TTC. For the Critical SSMs correct and timely identification of danger and activation of emergency braking is desirable. The time of activation of the AEB for every experiment was identified in the data. The critical safety metrics CFS and TTC were evaluated at that instance. The capability of each metric to predict the outcome was investigated. Afterwards, for the cases were the collision was not avoided, the crash severity was assumed to be relative to the speed of the vehicle at the moment of collision. That was correlated to the CFS core and the TTC value, to examine any possible indications of accident severity.

## 6. Results

### 6.1. Proactive fuzzy surrogate safety metric

From the synthetic emerging braking, every instance of car following is classified as safe or unsafe. As already mentioned, this resulted into 255,874 cases classified as safe and 1,510 classified as unsafe. Then all the instances were evaluated with the use of PFS, RSS 1, RSS 2, APB 1 and APB 2. The rate of true positives and true negatives for each model is presented in Table 2. The models RSS 2 and APB 2, using $a_{min,brake}$ equal to 3 m/s$^2$ are shown to be very conservative, identifying unsafety on the larger part of the experiment. RSS 1 and PFS 0.95 models had a small false positive rate of 0.02 % and 0.87 % respectively. However, there existed a large amount of false negatives, classifying a situation to be safe while in fact it was unsafe, as only 72.85 % unsafe cases were correctly classified by the RSS 1 and 94.97 % by the PFS 0.95. This is related to the fact that both metrics do not take into account the jerk on the deceleration and underestimate the effective safety distance.

Finally, APB 1 correctly classified all the unsafe situations while it misclassified 2.59 % of the safe ones, as the maximum jerk is taken into account. It must be noted that the model of the simulation of the stopping distance used to create the benchmark of safe and unsafe situations is very similar to the logic used for APB 1. So, the results of APB are not very representative. Also, the relatively high amount of false positives is due to the fact the RSS stopping distance assumes the vehicle accelerating with its maximum acceleration during the reaction time, which is not always the case. Additionally, according to the APB system, a vehicle that finds itself in unsafe distance must decelerate at least with $a_{max,brake}$, so even with APB 1 the vehicle should decelerate with 9 m/s$^2$. This would be more similar to the AEB and not really proactive. Otherwise, according to the APB 2 it would decelerate comfortably for too many cases. For any other parameter value between 3 and 9 m/s$^2$ there will be a tradeoff between decelerating very hard or very often.

Situations may be misclassified because of small differences on the reaction, as in the case for PFS 0.95, or because of the lack of jerk, as in the case of RSS 1, or due to unrealistic accelerations during reaction time as for the APB 1. This shows some of the problems related to the use of safety metrics adopting crisp thresholds. The proposed PFS metric is fuzzy to avoid the use of thresholds such as the one used above. In Fig. 3 all the situations are classified according to the deceleration needed by the rear vehicle to avoid an accident in case of hard deceleration of the front vehicle. The different distributions respectively refer to the cases where deceleration of 3 m/s$^2$ is sufficient, when the deceleration needed is between 3 m/s$^2$ and 6 m/s$^2$, between 6 m/s$^2$ and 9 m/s$^2$ and finally over 9 m/s$^2$ which are the most dangerous conditions. It is shown that even without taking into account the jerk, the majority of the really dangerous situations were identified to be unsafe with degree of truth more than 0.935, with few outliers going down to 0.88 (rightmost distribution in the Figure). Totally safe situations where mostly identified as such, with outliers going up to 0.2 (leftmost distribution in the Figure). In the two cases where not extremely hard accelerations are needed, it is shown that again PFS provides a meaningful comparison, with the milder cases having median value of 0.53 and maximum of 0.77, while the harder ones had median value of 0.86 and minimum of 0.70. Moreover, this result is in agreement with the diamond shaped severity hierarchy proposed by Svensson (Svensson (1998)). The events of absolute safety are not the most frequent in driver interactions, as drivers accept “medium severity” risks, to facilitate traffic flow.

This attribute of the PFS can be very useful. The reaction of the system doesn’t have to be too hard and the system does not need to react too often as the reaction can be scaled as well, imposing decelerations proportional to the PFS degree of truth, or even bounding the maximum acceleration for situations that are identified as unsafe with a small degree of truth.

### 6.2. Critical fuzzy surrogate safety metric

For each case, the instant in which the AEB started to decelerate was identified and both CFS and TTC were calculated. In Fig. 4 for each case, the CFS and TTC values are presented. The x-axis represents the maximum deceleration achieved by the vehicle. The green dots represent the tests in which the vehicle managed to stop before reaching the target, while the red “x” markers represent the tests in which the vehicle collided with the static target. The CFS correctly predicted all cases where there was a contact with the static target, having a membership value of 1, using only information at the beginning of the deceleration. Moreover, there have been cases with CFS membership value of 1 or very close, for which the vehicle managed to stop before reaching the target, but for all of them the vehicle managed to reach decelerations harder than 9 m/s$^2$, which was the assumed value. Furthermore, from the corresponding diagram of the TTC values at the time of the AEB activation, no such correlation is observed, as there have been cases where the deceleration started at TTC 0.4 s and the contact was avoided, while for cases when the deceleration started with TTC more than 0.8 s it wasn’t avoided.

**Table 2**
Accuracy for proactive SSMs on classifying the trajectory data.

| | Cases | RSS 1 | RSS 2 | APB 1 | APB 2 | PFS 0.95 |
|---|---|---|---|---|---|---|
| True Negative Rate | 255874 | 99.98 % | 13.93 % | 97.41 % | 15.76 % | 99.13 % |
| True Positive Rate | 1510 | 72.85 % | 100.00 % | 100.00 % | 100.00 % | 94.97 % |

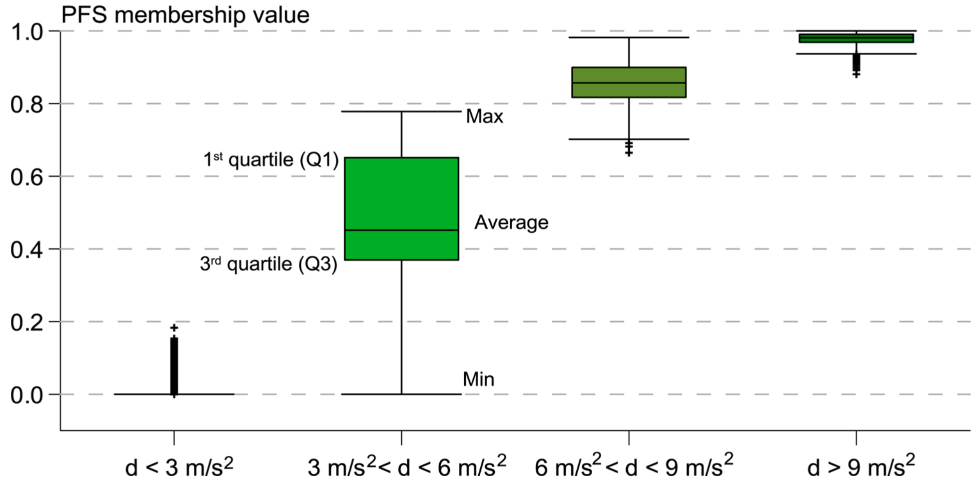


**Fig. 3.** PFS membership value for different cases of decelerations needed.

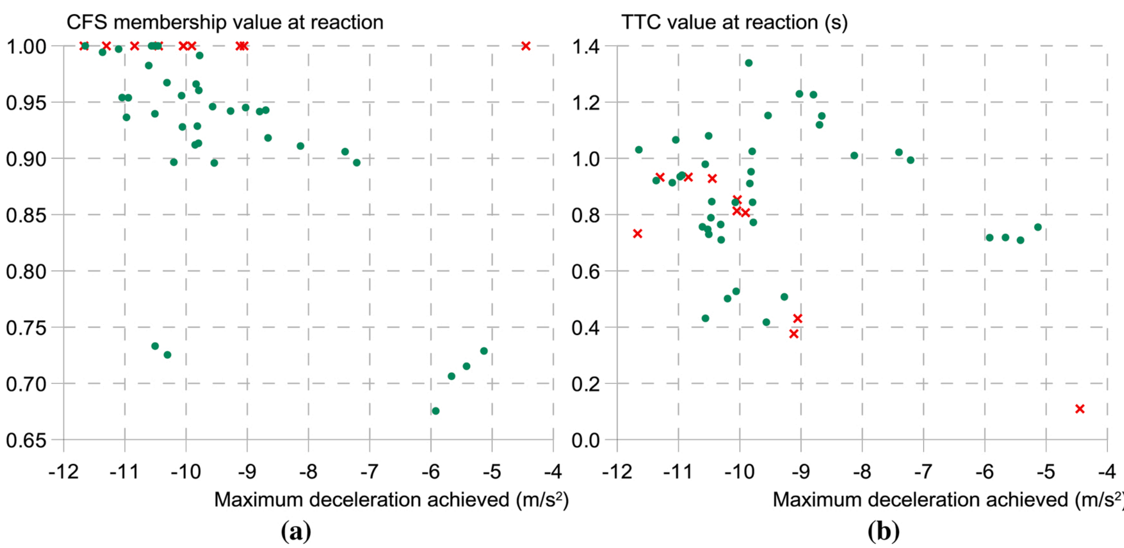


**Fig. 4.** Value of CFS (a) and TTC (b) at time of AEB activation to maximum deceleration of each experiment. Green dots represent the tests where the vehicle was able to stop before reaching the target while red crosses represent the ten cases in which the vehicles collided with the target (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article).

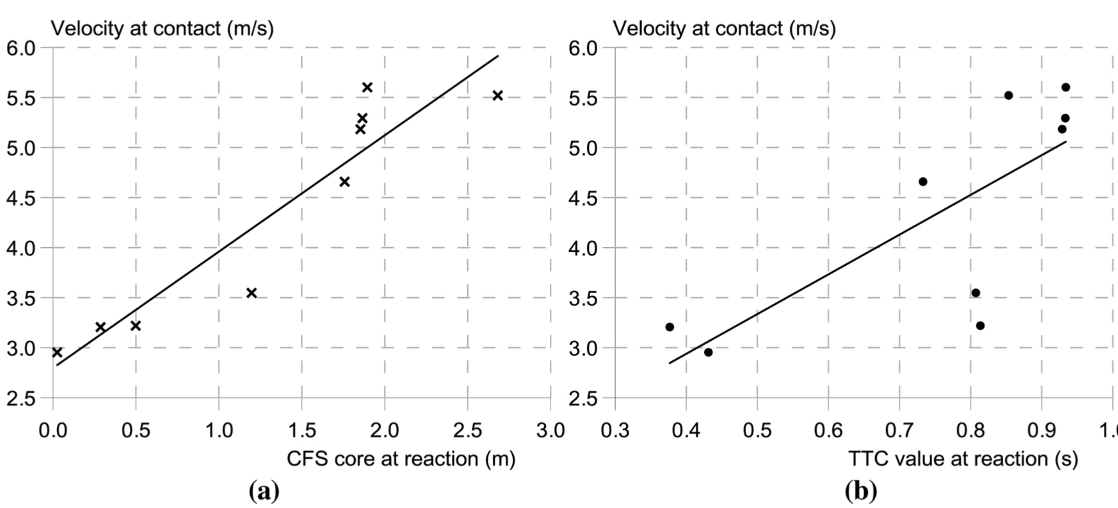


**Fig. 5.** Vehicles velocity at contact scatter plot with the CFS core (a) and with the TTC value (b) at time of AEB activation.

Furthermore, the fuzzy interval developed and used as a safety metric, apart from the membership value, has other attributes as the core and the support that hold information about the severity of a potential conflict. For nine out of the ten cases of the vehicle colliding with the static target, the velocity at the time of contact can be gathered from the data (as already mentioned, in one of the cases the driver steered away from the target to avoid the collision). For each of the nine cases, the diameter of the core of the CFS calculated at the first instance of deceleration is correlated to the velocity of the vehicle at the moment of the collision. Results are reported in Fig. 5 (a). Fig. 5 **(b)** displays the respective velocity at contact over the TTC value at the time of AEB reaction. For the TTC, the points are more scattered and in any case the pattern is counterintuitive because for the highest collision velocities the TTC was larger, indicating less dangerous situations. The correlation between the CSF core and the collision speed provide an important insight that can be used to further assess the safety of automated driving systems.

## 7. Conclusions

The present paper introduces two fuzzy Surrogate Safety Metrics (SSMs) for rear-end collision, the Proactive Fuzzy Surrogate Safety Metric (PFS) and the Critical Fuzzy Surrogate Safety Metric (CFS) and assess their performance. The distinction is based on two different definitions of safety. Critically unsafe is considered to be a condition in which an evasive maneuver is required in order to avoid an accident. Proactively unsafe is considered to be a situation in which if the front vehicle starts an emergency deceleration, the rear vehicle will not have enough space to perceive and react to the danger without an accident. The fuzzy SSMs have been proposed and investigated on synthetic data in (Mattas et al., 2019b).

The study investigates the applicability of the two metrics for assessing the safety level of automated functionalities or AV controllers, both during system development and controller design. To achieve this goal, an experimental campaign was organized in the AstaZero proving ground in Sweden. The campaign consisted of two main parts: a car-following experiment including five vehicles solely driven by their Adaptive Cruise Control (ACC) system and a number of experiments to test the performance of the Autonomous Emergency Braking (AEB) system implemented in the vehicles to avoid collision with a static target. Additional tests were conducted to measure the hardest possible deceleration of each vehicle, imposed by a human driver.

The first part was used to assess the performance of the PFS, in comparison with four different implementations of the Responsibility Sensitive Safety model. Results show that adopting crisps thresholds to classify safe and unsafe conditions can produce reasonable results. However, there is always a non-negligible amount of false positive and false negative classifications, leading to problems if used for vehicle controllers. On the contrary, PFS shows superior classification capabilities, and its characteristics based on fuzzy logic would not require the vehicle to decelerate very sharply or very often.

The second part of the campaign was used to assess the performance of the CFS in comparison with the TTC. The CFS was able to recognize all 10 occurrences in which the vehicles collided with the target, while the TTC was not. Moreover, the CFS demonstrated additional attributes as its core holds information about the severity of a potential conflict. Both features make the CSF suitable to assess the safety of advanced driving assistance and automation systems.

In the future, further investigation of the CFS and PFS attributes on real-life data should be carried out, using also moving targets. Moreover, an investigation on the use of the maximum jerk in the calculation of CFS and PFS can be carried out, as it has been observed that it has significant influence on the stopping distance. Finally, a fuzzy controller can be developed using PFS and CFS for carrying out different automated functionalities safely and efficiently.

## CRediT authorship contribution statement

**Konstantinos Mattas:** Conceptualization, Methodology, Writing - review & editing, Visualization, Software, Writing - original draft, Investigation, Data curation. **Michail Makridis:** Conceptualization, Methodology, Writing - review & editing, Software, Writing - original draft, Investigation, Data curation. **George Botzoris:** Conceptualization, Methodology, Writing - review & editing, Writing - original draft, Visualization. **Akos Kriston:** Software, Writing - original draft, Investigation, Data curation. **Fabrizio Minarini:** Software, Writing - original draft, Investigation, Data curation. **Basil Papadopoulos:** Conceptualization, Methodology, Writing - review & editing, Writing - original draft. **Fabrizio Re:** Software, Writing - original draft, Investigation, Data curation. **Greger Rognelund:** Software, Writing - original draft, Investigation, Data curation. **Biagio Ciuffo:** Conceptualization, Methodology, Writing - review & editing, Software, Writing - original draft, Investigation, Data curation.

## Declaration of Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to the influence the work reported the work reported in this paper.

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