Running to Keep in the Same Place: Consumer Choice as a Game of Status

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If individuals care about their status, defined as their rank in the distribution of consumption of one "positional" good, then the consumer's problem is strategic as her utility depends on the consumption choices of others. In the symmetric Nash equilibrium, each individual spends an inefficiently high amount on the status good. Using techniques from auction theory, we analyze the effects of exogenous changes in the distribution of income. In a richer society, almost all individuals spend more on conspicuous consumption, and individual utility is lower at each income level. In a more equal society, the poor are worse off. (*JEL* C72, D11, D31, D62)

> Now *here,* you see, it takes all the running
> you can do to keep in the same place.
> —Lewis Carroll (1871),
> *Through the Looking-Glass*

Neoclassical economic theory assumes that an agent's utility depends solely on the absolute level of personal consumption. An alternative assumption, that utility or happiness depends at least in part on the comparison of one's own consumption to that of others, dates back at least to Thorstein Veblen's seminal work of 1899, and was first formally modelled by James S. Duesenberry (1949). More recently, compelling evidence has accumulated that people tend to evaluate their own consumption in the light of the consumption of others. For example, starting from Richard Easterlin (1974), a number of studies have found that self-reported happiness may be more sensitive to relative than to absolute income.[1] There is also now a developing theoretical literature that examines the implications of the presence of relative concerns in agents' preferences.

In this paper, we take a new approach that emphasizes the strategic nature of concerns for relative position and analyzes their interaction with the distribution of income. Assume that an agent's status depends on her consumption relative to that of others. Assume further that her utility depends at least in part on her status. Then the choice of levels of consumption is necessarily strategic, because each agent must anticipate the consumption decisions of others in making her optimal consumption decision. Here we model concern for status, as indicated by ordinal rank in the distribution of consumption, as a simultaneous move game. In the symmetric Nash equilibrium, an individual's position in the distribution of consumption coincides with her position in the distribution of income. That is, everyone increases conspicuous consumption in order to improve status, but any gain in status is cancelled out by the similarly increased expenditure of others. Such an economy can be described as a Lewis Carroll

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[1] For a survey of empirical work on happiness see Andrew J. Oswald (1997). Further empirical evidence on the importance of relative concerns can be found in Robert H. Frank (1985a); Huib van de Stadt et al. (1985); Andrew E. Clark and Andrew J. Oswald (1996); David Neumark and Andrew Postlewaite (1998); Sara J. Solnick and David Hemenway (1998).

"Red Queen" economy, in which "it takes all the running you can do to keep in the same place."[2]

Furthermore, treating status strategically allows us to analyze how exogenous changes in income distribution can affect individual choices. On a formal level, for the form of preferences that we analyze, the problem of the consumer is very similar to that of a bidder participating in a first-price auction. We are able to employ techniques from auction theory to show that as income in society increases, the "Red Queen" effect becomes more significant: the proportion of income spent on conspicuous consumption increases and equilibrium utility falls at each level of income. Partly this is because, as a society becomes richer, those whose incomes do not grow spend more on conspicuous consumption in an attempt to keep up.[3] Second, we show that if income becomes more equally distributed (in a sense of second-order stochastic dominance, or, equivalently, generalized Lorenz dominance), the utility of those with fixed low incomes falls. Finally, we consider some policy implications of the model and find that a suitable consumption tax and subsidy scheme can be welfare improving. Perhaps surprisingly, however, if the income distribution changes in the direction of greater equality, the marginal rate of tax and subsidy for those with middle incomes should rise, and for those with high incomes the marginal rate should fall.

The idea that those with low incomes might lose from greater equality is somewhat surprising, but it reflects the old phrase "misery loves company." Consider those with low income who are left behind when others' incomes are raised as consequences of an increase in overall equality. These people now see fewer people with similar or lower incomes. Furthermore, they observe the increased consumption expenditure of those who have benefited from the change in income distribution. There is therefore social pressure to raise their own consumption levels. In contrast, at the top of the income distribution an increase in equality will reduce the competition for status as it thins the ranks of the rich. Consequently, the rich may gain from an increase in equality. The effect of increased equality on happiness of an entire society is ambiguous. This is because a change in income distribution has two effects—on one hand, it leads to a change in individual utility, but on the other hand, it changes the composition of society. Simply put, those at the bottom are worse off in a more equal society, but at the same time there are fewer of them. Thus, when greater equality leads to a happier society, it is only because an increase in a mass of richer (thus happier) people offsets the unhappiness of the have-nots.

These results appear to be particularly timely in that the relationship between happiness, income, and inequality has been subject to much recent empirical work. Indeed, there is some empirical support for our finding that greater equality does not necessarily lead to greater happiness. Clark (2003), using British panel data, finds a positive relationship between inequality and self-reported happiness while Claudia Senik (2003) finds that inequality has no statistical influence on life satisfaction in post-reform Russia. On the other hand, Alberto Alesina et al. (2003) report a negative relationship between inequality and happiness for both Europe and the United States. Interestingly, they find that in the United States it is only the rich who are bothered by recent income inequality, while in Europe only the poor are found to be unhappy with inequality. The above empirical studies offer a range of possible explanations for the relationship between inequality and happiness, drawing upon diversity of social norms, informational content, and dynamic social mobility, all not captured by our static model. However, one result that is consistent across these studies is that relative income seems to matter for self-reported happiness, an observation first made by Easterlin (1974).

Assuming that people care about their relative position leaves unanswered how exactly such preferences should be modelled. Indeed, we can divide the existing literature on this topic into two strands. The first approach, stemming from Duesenberry (1949) and Robert Pollack (1976), employs interdependent prefer-

[2] The idea of using Lewis Carroll's Red Queen as a metaphor for competition has already been used in evolutionary biology and in evolutionary game theory.

[3] As we make clear in Section II, our analysis concerns changes to utility at a fixed level of income, not at a fixed relative position or quantile.

ences represented by utility functions that depend not only on the absolute value of consumption, but also on the average level of consumption (referred to as "the Joneses") within a population.[4] In this paper, however, we concentrate on the other formulation of interdependent preferences that involves concern with one's status, as indicated by the ordinal rank in the distribution of consumption but also potentially income or wealth. It was pioneered by Frank (1985b) in a study of the demand for positional and nonpositional goods. Arthur Robson (1992) considers preferences over ordinal rank in wealth as well as absolute wealth. Alexis Direr (2001) considers preferences over absolute and relative consumption in each of two periods of the lifetime of an individual. Ben Cooper et al. (2001) analyze a growth model where there is concern for status, but agents only interact with other agents of the same wealth, so there is no interaction between consumption choices and the distribution of income as there is here.

The possible reasons why people may possess a concern for status are also diverse. First, this type of preference may be intrinsic or "hard-wired," a fundamental human characteristic. Many economists would be happier with the alternative possibility that agents can have an "instrumental" concern for status, that is, they do not value status itself but seek it because high status allows better consumption opportunities. This second approach has been advocated by Andrew Postlewaite (1998). Consumption and saving decisions when status is instrumental were studied extensively by Harold L. Cole et al. (1992, 1998). In their model, status as indicated by ordinal rank in the distribution is instrumental to a (marriage) matching. That is, those with higher rank have higher consumption because they marry better. Our model is consistent with either underlying motivation.

Conspicuous consumption has also been modelled in terms of signaling. This again can have either an instrumental or psychological justification. For example, conspicuous consumption may serve to signal desirability as a marriage partner as it may indicate wealth that is otherwise unobservable (Cole et al., 1995; Corneo and Jeanne, 1998). In the models of Norman J. Ireland (1994, 2001) and Laurie Simon Bagwell and B. Douglas Bernheim (1996), however, agents engage in conspicuous consumption as a means to signal one's wealth as there is intrinsic satisfaction in being viewed as prosperous. Since in these models resources are diverted to signaling, equilibria are typically not Pareto efficient, something true in Frank's original analysis (1985b) and the model considered here. A more subtle, but still important, point is that in the model considered here, an agent's payoffs depend on others' perception of her *relative* position, whereas in Ireland (1994, 2001), for example, an agent's payoffs depend on others' perception of his absolute level of consumption.

Thus, there is a considerable existing literature both on conspicuous consumption and relative concerns. What is distinctive about our approach is the analysis of the interaction between the distribution of income, status, conspicuous consumption, and welfare. To our knowledge, this has not been explored before. The paper is organized as follows. Section I introduces the game of status and shows the existence of a unique symmetric equilibrium. Section II shows how comparative statics predictions can be obtained on equilibrium utility and consumption behavior. Section III explores the possibilities of a corrective consumption tax. Section IV concludes.

I. A Game of Status

Consider the problem of a consumer who must decide how to divide her income between the purchase of two different goods. The neoclassical solution to this problem can be found in any textbook on microeconomics. However, now imagine that expenditure on one of the two goods provides some form of status which gives an agent utility distinct from the direct utility from its consumption. For example, if the Joneses buy a large car, this may arouse the envy and admiration of their neighbors, pleasing the Joneses. Suppose in particular that this status arises from relative and not absolute levels of consumption. That is, it is not just that the car is

[4] Further works in this growing literature include Andrew B. Abel (1990); Jordi Galí (1994); Richmond Harbaugh (1996); Giacomo Corneo and Olivier Jeanne (1997); Clark and Oswald (1998).

big but that it is bigger than those owned by the neighbors that also matters. Then, first, the good is *positional* in the sense of Fred Hirsch (1976). Second, the standard problem of choosing consumption levels becomes a game between consumers. Individuals will engage in competition in terms of conspicuous consumption of the positional good, that is, in a game of status.

There may be various reasons why people care about status. An agent's position in society can enter his utility function for conventional economic reasons, as Postlewaite (1998) suggested. That is, individuals may care about their status mostly "instrumentally," as societies frequently allocate goods according to one's status rather than through markets. An example is when marriage arrangements are such that one's status determines possible marriage partners. Conspicuous consumption bestows status and thus allows better marriage opportunities as it signals income or productivity that is otherwise unobservable.[5] However, signalling is not the only explanation. As Veblen (1899) and later Duesenberry (1949) argued, people may aspire for higher status as an end in itself. Individuals gain psychological satisfaction from being better off than others and feel uneasy when they see others doing better. There is no need for incomplete information to produce conspicuous consumption. For example, the Smiths may know the Joneses are richer than the Smiths, however, they may not envy them unless they visibly see the Joneses enjoying a more lavish lifestyle. To be more specific, let $F(\cdot)$ be the distribution of the consumption of the positional good in society. For an individual whose own level of conspicuous consumption is x, $F(x)$ gives the expected frequency with which she will be able to make these pleasurable favorable comparisons in terms of visible prosperity between herself and another individual. Her utility will then be increasing in $F(x)$.

We do not try to adjudicate between these two possible explanations as to why people may have relative concerns.[6] Instead, we analyze the behavior of agents possessing such preferences. This is done in the context of a simultaneous move game of incomplete information. We assume an economy consisting of a continuum of agents, identical except in terms of income. Each agent is endowed with a level of income z which is private information and is an independent draw from a common distribution.[7] This is described by a distribution function $G(z)$ which is twice continuously differentiable with a strictly positive density on some interval $[\underline{z}, \bar{z}]$ with $\underline{z} \geq 0$.

We follow Frank (1985b) and assume that each agent must choose how to allocate his income between a visible (positional) good which carries status and another (nonpositional) good, the consumption of which is not directly observable by other agents. Let x be the amount consumed of the positional good, and y the amount of the nonpositional good. We will refer to consumption of x as conspicuous consumption. A strategy for an agent will be a choice of a mapping from income z to consumption x. Agents' choices of conspicuous consumption are aggregated in a distribution of conspicuous consumption $F(\cdot)$, with $F(x)$ being the mass of individuals with consumption less than or equal to x. Following Frank (1985b) and Robson (1992), an agent's status will be determined by her position in this distribution of conspicuous consumption. Here, we define status as follows:

$$(1)\quad S(x, F(\cdot)) = \gamma F(x) + (1-\gamma)F^{-}(x) + \alpha$$

where $\gamma \in [0, 1)$ and $F^{-}(x) = \lim_{x' \to x^{-}} F(x')$ is

[5] Marriage matching for a continuum population has been extensively explored in Cole et al. (1992, 1995, 1998). The link between matching, status, and conspicuous consumption under incomplete information is studied by Corneo and Jeanne (1998). For a finite population, Jan Eeckhout (2000) demonstrates that when each sex can rank each member of the other according to a common criterion, which here we would take to be status, the only stable voluntary matching mechanism is the one in which each woman is matched to a man whose rank in the distribution of males is equal to her rank in the distribution of females.

[6] Indeed, they are not mutually exclusive. Perhaps it is because such marriage matching problems were important for our ancestors that we have preferences for status now. Mari Rege (2001), for example, finds that a concern for social status can be evolutionarily stable in a matching market.

[7] We assume that income is private information to allow for the instrumental, signaling story behind our model. As argued above, under the alternative psychological justification, even if income is common knowledge people may engage in conspicuous consumption to promote feelings of self-worth.

the mass of individuals with consumption strictly less than x.[8] The parameter $\alpha \geq 0$ is a constant representing a guaranteed minimum level of status.[9]

Our definition of status is a small modification of the one suggested by Robson (1992). An alternative simpler specification is for an agent choosing conspicuous consumption x to have status $F(x)$, as in Frank (1985b). Frank's specification, however, is prone to the following problem.[10] If all agents chose the same level of conspicuous consumption, $\tilde{x}$, then, as $F(\tilde{x}) = 1$, all agents would have the highest possible status. Since all individuals have zero weight given the infinite population, if an individual increases x above $\tilde{x}$, she would see no increase in status. On a technical level, this may result in nonuniqueness of the equilibrium of the game. It is also more plausible that being uniquely first is more attractive than being equal first. In our specification of status this is captured by the parameter γ which is a constant representing a decrease in satisfaction resulting from "ties." It implies that if all agents were to choose $\tilde{x}$, they would have status $\gamma + \alpha$, while the status of an individual switching to $x > \tilde{x}$ would be $1 + \alpha$, that is, strictly greater. Note that the distribution $F(\cdot)$ will be determined endogenously by the choices of the agents, and as we will see, given this specification of status, it will have no mass points in equilibrium.

We further assume that all agents have the following utility function which can be decomposed into two elements. The first $V(x, y)$ is a "conventional" utility function, that depends only on one's own consumption. The second is the measure of status defined above.

$$(2) \quad U(x, y, S(x, F(\cdot))) = V(x, y)S(x, F(\cdot)).$$

We assume that $V(\cdot)$ is nonnegative, strictly increasing in both its arguments, strictly quasiconcave and twice differentiable. We further assume that $V_{ii} \leq 0$ for $i = 1, 2$ and that $V_{ij} \geq 0$ for $i \neq j$.[11] The assumption that the status term $S(x, F(\cdot))$ enters multiplicatively into the preferences equation (2) brings out the formal resemblance of the problem to a first-price sealed-bid auction. This is strongest when $\alpha = 0$, where it is as though we have a bidder who gains a utility $V(x, y)$ if she wins with a bid x, and $F(x)$ is the probability of winning. Increasing one's expenditure on the positional good leads to a trade-off between the increase in status and the lowering of direct utility from decreased consumption of the nonpositional good, just as a bidder in an auction must trade off increasing his probability of winning against lower realized profits in the event of winning. It is this formal resemblance to an auction that permits clear comparative statics results.[12] In any case, each agent faces the following problem,

$$(3) \quad \max_{x,y} V(x, y)(\gamma F(x) + (1 - \gamma)F^{-}(x) + \alpha)$$

$$\text{subject to } px + y \leq z, x \geq 0, y \geq 0$$

where p is the price of the positional good. The price of the nonpositional good is normalized to one.

In this context, a *symmetric equilibrium* will be a Nash equilibrium in which all agents use the same strategy, that is, the same mapping $x(z)$ from income to expenditure. Suppose for

[8] Functions F and F^{-} are standard in the theory of random variables. As Patrick Billingsley (1995, pp. 187–89) points out, for any distribution function F, the function F^{-} is well defined.

[9] In the instrumental interpretation of status due to Cole et al. (1992) discussed earlier where high status leads to a good match, α would represent the value of the least attractive match. The other interpretation of α is purely technical. Our utility function is basically multiplicative in status. Introducing α introduces an element of additivity to the utility function and serves as a check on the robustness of our results. See also the discussion following Lemma 1 below.

[10] We are grateful to Daniel Quint for bringing this to our attention.

[11] We will use these assumptions (with the last one similar to Eric Maskin and John Riley's (2000) "weak supermodularity" assumption), to ensure that optimal strategies are weakly increasing—see Lemma A1 in the Appendix.

[12] As in an auction, utility will be discontinuous in the actions of others. For example, if a mass of people with consumption less than that of some individual i increase their consumption this will have no effect on the utility of i until they surpass her, when there will be a downwards jump in her utility. As discussed earlier, Duesenberry (1949) introduced an alternative model of relative concerns where an agent's utility depends smoothly on the mean consumption of others, and there would be no such discontinuities. However, this model lacks an instrumental justification similar to that underlying status as rank, and does not afford the same tractability. That is, the discontinuities in the present model actually aid analysis.

the moment that the equilibrium strategy $x(z)$ is strictly increasing and differentiable. We will go on to show that such an equilibrium exists. Assume further that all adopt the equilibrium strategy. Then, the probability that an individual i with income level z_i and consumption choice x_i has higher consumption than another arbitrarily chosen individual j is $F(x_i) = F^-(x_i) = \Pr(x_i > x(z_j)) = \Pr(x^{-1}(x_i) > z_j) = G(x^{-1}(x_i))$. Note that, given our definition (1), the individual's status is now determined by her position in the distribution of income, or $S(x_i, F(\cdot)) = G(x^{-1}(x_i)) + \alpha$. Thus, we can write i's utility as $V(x_i, z_i - px_i)(G(x^{-1}(x_i)) + \alpha)$, substituting for y using the budget constraint. Differentiating with respect to x_i, the resultant first-order conditions can be written as

$$(4)\quad (V_1(x_i, z_i - px_i) - pV_2(x_i, z_i - px_i)) \times (G(x^{-1}(x_i)) + \alpha) + V(x_i, z_i - px_i) \times \frac{g(x^{-1}(x_i))}{x'(x^{-1}(x_i))} = 0.$$

Given that in a symmetric equilibrium we have $x_i = x(z_i)$, an agent's position $F(x)$ in the distribution of consumption will be equal to his rank $G(z)$ in the distribution of income and the first order conditions are now

$$(5)\quad V_1(x, z - px) - pV_2(x, z - px) + V(x, z - px)\frac{g(z)}{x'(z)(G(z) + \alpha)} = 0.$$

Note that the first two terms in (5) are the first-order conditions for the standard consumer problem. Now there is an additional term that represents the additional marginal return to expenditure on x due to enhanced status. The equation (5) implies the following first-order differential equation:

$$(6)\quad x'(z) = \left(\frac{g(z)}{G(z) + \alpha}\right)\left(\frac{V}{pV_2 - V_1}\right) = \frac{g(z)}{G(z) + \alpha}\,\psi(x, z).$$

We will show that this differential equation has a unique solution, which will form a symmetric equilibrium of the game of status.

But first we need to define the equivalent of what Frank (1985b) calls the *cooperative case*. This will prove useful as a point of comparison. It simply assumes that each agent makes a consumption choice (x_c, y_c) according to the standard tangency condition. That is,

$$(7)\quad V_1(x_c, y_c)/V_2(x_c, y_c) = p.$$

Let $x_c(z)$ be the strategy implied by the above condition. The cooperative strategy also enables us to fix the appropriate boundary condition for the differential equation (6). This is not a purely technical question. As we will see, equilibrium behavior is quite different in the two different cases, when α minimum guaranteed status is zero, and when it is positive.

LEMMA 1: *In a symmetric equilibrium in strictly increasing strategies,*

(*i*) *if* $\alpha = 0$, *then* $x(\underline{z}) \leq \underline{z}/p$ *with* $\lim_{z \to \underline{z}^+} x(z) = \underline{z}/p$;
(*ii*) *if* $\alpha > 0$, *then* $x(\underline{z}) = x_c(\underline{z})$.

In a symmetric equilibrium, the individual with lowest income will have the lowest status, that is, equal to α. The question in effect is what is the appropriate competitive response in a contest where you know you will come last. Now, when α is zero, given the utility function assumed in (3), the individual with lowest status will always gain zero utility and is therefore indifferent over any level of x between 0 and $\underline{z}/p$. Furthermore, individuals with low income are "desperate" to avoid zero status and as the lowest level of income is reached, the proportion of income spent on the conspicuous good x approaches 100 percent.[13] In contrast, when α is positive, so the consequences of coming last are

[13] See Veblen (1899, p. 85): "No class of society, not even the most abjectly poor, foregoes all customary conspicuous consumption. The last items of this category of consumption are not given up except under stress of direct necessity. Very much of the squalor and discomfort will be endured before the last trinket or the last pretence of pecuniary decency is put away." For further discussion of conspicuous consumption by the poor see Luuk van Kempen (2003).

not so severe, there is less pressure to compete and the individual with the lowest income behaves as though she were completely indifferent to status concerns and spends the cooperative amount.[14]

One can associate these two cases with two different pictures of conspicuous consumption. In the first, more conspicuous consumption, as a percentage of income, is carried out by those least able to afford it. As indicated above, we have in mind that social rank may have a significant impact on marriage prospects with, at least in our evolutionary past, low rank often leading to failure to reproduce. In an evolutionary context, this is the lowest payoff possible. Even today, one could argue that very low social status is associated with unemployment, poor marriage prospects and social exclusion. It is in this context that it is possible that even people with very low incomes may have strong incentives to increase their status and the assumption of zero α may be plausible. In the second, a positive value for α implies that the poorest individual has a guaranteed level of status independent of the consumption behavior of others and conspicuous consumption is largely limited to the middle and upper classes, the "gentlemen of leisure" in Veblen's (1899) terminology. If one believes that modern society is not quite as cutthroat as in the past, such an assumption might be more appropriate. What is surprising, however, is that many of our findings are robust across the two different specifications, including this important result.

PROPOSITION 1: *The unique solution to the differential equation* (6) *with the boundary conditions*

$$x(\underline{z}) = \underline{z}/p \text{ for } \alpha = 0 \tag{8}$$

$$x(\underline{z}) = x_c(\underline{z}) \text{ for } \alpha > 0 \tag{9}$$

is an essentially unique symmetric Nash equilibrium of the game of status. Equilibrium conspicuous consumption $x(z)$ *is greater than the cooperative level,* $x(z) > x_c(z)$ *on* $(\underline{z}, \bar{z}]$, *and* $x(z)$ *is continuous and strictly increasing in income z so that rank in the positional good is equal to rank in income, that is,* $F(x) = G(z)$.

The essence of the proof is to verify that the equilibrium strategy will be strictly increasing and differentiable. It must then satisfy the differential equation (6), to which there is only one solution that satisfies the boundary conditions in Lemma 1. In the terminology of Bernard Lebrun (1999), the equilibrium is thus "essentially" unique, in that it is unique on the interval $(\underline{z}, \bar{z}]$. As we have seen, in the case of $\alpha = 0$, these boundary conditions do not precisely specify the equilibrium behavior of agents with the lowest income level $\underline{z}$. We concentrate on the case where they choose $x(\underline{z}) = \underline{z}/p$ as this maintains the continuity of the equilibrium strategy and therefore simplifies the exposition.[15]

We can also make the following observation about Nash equilibrium behavior, first made by Frank (1985b). Suppose that status, instead of being endogenously determined, was exogenously given as one's rank in the distribution of income, that is, $S = \alpha + G(z)$. That is, richer agents have higher status. It is easy to verify that, as in these circumstances an agent cannot affect her status by her choice of consumption, each agent will maximize her utility by choosing the cooperative level of consumption $x_c(z)$. The result in Proposition 1, that spending on conspicuous consumption in the noncooperative equilibrium is higher than in the cooperative case, clearly follows from a comparison of (7) with the noncooperative first order conditions (5). Yet in both cases, an individual's status is exactly determined by his position in the original income distribution. In the equilibrium, the additional expenditure on conspicuous consumption has no net effect on the individual's position in the social hierarchy, and thus it is

[14] Mathematically speaking, there is a discontinuity in the behavior of low-income individuals between the case when α is zero, and when it is arbitrarily small (unless the lowest income $\underline{z}$ is zero, in which case the poorest agent must spend zero in both cases).

[15] The fundamental mathematical problem is the potential singularity in the differential equation (6) as $G(\underline{z}) = 0$. This has been a difficulty for auction theorists for decades, as in some cases it gives rise to multiple, asymmetric equilibria. Our current result does not rule out asymmetric equilibria. However, as the game of status considered here is strategically similar to a symmetric first-price private value auction for which there are no asymmetric equilibria [see Lebrun, 1999, Corollary 3(v)], we conjecture that there are no asymmetric equilibria here either.

"wasteful" in the sense it leads to a Pareto-inferior outcome.[16] If all agents could agree to stick to the cooperative solution, everyone would be better off. But this is not a Nash equilibrium.

As one interpretation of the role of conspicuous consumption in the present model is as a signal of unobservable income, one might wonder why the only equilibrium here is separating, when signaling models so often also have pooling equilibria. On a general level, pooling equilibria exist if those seeing higher signals do not interpret this as evidence that the senders are higher types. Here, however, our definition of status, given in (1) above, always assigns higher status to an agent whose consumption is higher than another. This gives an incentive to increase x to break any ties.[17] A second important feature of the current model is that, in contrast to other signaling models of conspicuous consumption such as Bagwell and Bernheim (1998) or Ireland (2001), payoffs depend on the perceived relative position rather than the perception of one's (absolute) type. In both types of model, a separating equilibrium involves a choice of conspicuous consumption that is strictly increasing in income, resulting in equilibrium status being increasing in income. However, the relative nature of payoffs here means that the equilibrium conditions here depend on the distribution of income in a way that they do not in other signaling models.

What is distinctive about the approach here is that it makes explicit [through the differential equation (6)] that individual consumption behavior depends on the distribution of income in society. While an explicit characterization of equilibrium is only possible for specific utility functions and income distributions, we can obtain quite general comparative static results on the effect of changes in the entire income distribution on equilibrium behavior and equilibrium utility. This is explored in the next section.

II. The Distribution of Income and Conspicuous Consumption

Treating status strategically not only seems to be a more reasonable approach but it also allows us to consider the effect of a change in the entire income distribution on consumer choice. From our analysis in the previous section, we know that equilibrium demand is given by a solution to the differential equation (6). To obtain an explicit solution to this differential equation, one has to place strong restrictions on the form of the utility function and the distribution function. Luckily, however, some comparative static analysis of the equilibrium consumptions decisions and utility is possible even without an explicit solution.

Consider two societies, A and B, which differ only in terms of income distributions given by cumulative distributions $G_A(z)$ and $G_B(z)$ respectively, both having support on $[\underline{z}, \bar{z}]$. We will consider how changes in the distribution of income affect conspicuous consumption and welfare. To do this, we need a way to order two income distributions, so that we can say one is "higher" than the other. The most common ordering of this kind is (first-order) stochastic dominance, which requires that if the distribution G_A stochastically dominates a different distribution G_B, then $G_A(z) \leq G_B(z)$ everywhere on $[\underline{z}, \bar{z}]$.

But we will also want to consider the effects of the redistribution as well as the growth of income. That is, we also need orderings that reflect changes in the level of inequality in society. Since the work of Anthony B. Atkinson (1970), second-order stochastic dominance has become a standard way in which to rank income distributions in terms of inequality. If a distribution of income G_A second order stochastically dominates distribution G_B then the generalized Lorenz curves of the two distributions do not

[16] This is true from the point of view of the participants in the game of status, but maybe not for society as a whole. Some forms of conspicuous expenditure, extravagant presents, lavish dinners for example, may represent transfers to those one is trying to impress. Indeed, as Amihai Glazer and Kai A. Konrad (1996) have argued, one way to signal one's wealth is to be seen to give to charity.

[17] We are grateful to Daniel Quint for presenting us with the following example of the existence of a pooling equilibrium for a different specification of status. Let $S(x, F(\cdot)) = F(x)$ (as in Frank, 1985b) and let $p = 1$, $\alpha = 0$, and $V(x, y) = xy$. Let G be any distribution on $[\underline{z}, \bar{z}] = [15, 20]$. For any $x^* \in (10, 15)$, a symmetric equilibrium exists where every player consumes x^* of the positional good as a strict best response. To see that, suppose everyone else is consuming x^*. Since $F(x) = 0$ for $x < x^*$ and 1 for $x \geq x^*$, a given player faces payoffs $U(x, z) = x(z - x)$ if $x \geq x^*$ and $U(x, z) = 0$ otherwise. This payoff is maximized by setting $x = x^*$, since by construction $x^* > z/2 = x_c$.

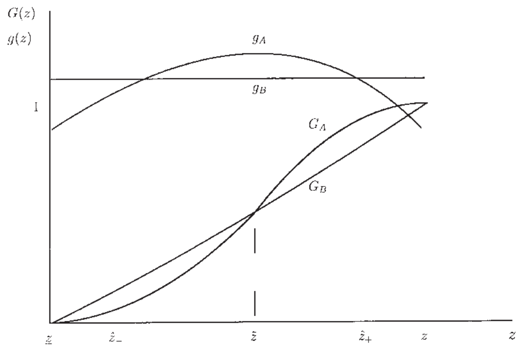

FIGURE 1. AN EXAMPLE WHERE DISTRIBUTION G_B IS A MEAN-PRESERVING SPREAD OF G_A

cross, a property known as generalized Lorenz dominance. Mathematically, G_A second order stochastically dominates G_B if and only if

$$(10)\quad \int_{\underline{z}}^{z} G_A(t)\,dt \le \int_{\underline{z}}^{z} G_B(t)\,dt \qquad \text{for all } z \in [\underline{z}, \bar{z}].$$

Clearly, this inequality will hold if G_A first order stochastically dominates G_B. However, it also holds for other cases, such as mean-preserving spreads (such as the one illustrated in Figure 1), which do not satisfy first-order dominance. Thus, if the income distribution of society A second order stochastically dominates that of society B, society A is either richer or more equal, or both, than society B.

We now turn to the comparative static results for equilibrium behavior and individual utility.[18] We first investigate the implications of a change in the distribution of income for welfare. Our results concern the utility of someone whose income remains unchanged as the distribution of income in society changes. There are two reasons why we take this approach rather than the alternative of examining utility at a fixed rank or quantile, for example, of the median individual. First, the current approach is relatively tractable. Second, it highlights the difference in the results obtained here from those obtained under neoclassical assumptions. In standard models, as an individual's utility depends only on her consumption, ceteris paribus, her utility would not be affected by changes in the incomes of others. Here, this is not the case and it happens for two reasons. First, a change in the distribution of income will alter an individual's equilibrium status $G(z) + \alpha$. Second, equilibrium expenditure on conspicuous consumption will change. As we will see later on in this section, an individual who suffers a fall in status as others' incomes rise may be forced to increase conspicuous consumption in an attempt to keep up, leading to yet lower utility.

We look at the effect of an "improvement" in the distribution of income in the sense of second-order stochastic dominance. That is, society A is more equal or more prosperous. If condition (10) holds, and if G_A and G_B are sufficiently distinct in a sense we make precise in Proposition 2 below, it must be that there exists some level of income $\tilde{z} > \underline{z}$ such that $G_A(z) < G_B(z)$ on $(\underline{z}, \tilde{z})$. In other words, all individuals with incomes below $\tilde{z}$ occupy a lower social position in a society A. An example of two distributions satisfying this requirement is illustrated in Figure 1. Let $U^*(z) = V(x^*(z), y^*(z))(G(z) + \alpha) = V^*(z)(G(z) + \alpha)$ be the individual utility gained in the symmetric equilibrium. Our first result concerns the welfare of those at the bottom of the distribution.

PROPOSITION 2: *Suppose that G_A second order stochastically dominates G_B and that there are a finite number of points in $[\underline{z}, \bar{z}]$ where $G_A(z)$ equals $G_B(z)$. Denote the first crossing as $\tilde{z}$. Then, for any $\alpha \ge 0$, $U_A^*(z) \le U_B^*(z)$ for all $z \in [\underline{z}, \tilde{z}]$.*

That is, the "poor," those with incomes in the interval $[\underline{z}, \tilde{z}]$, that is the bottom end of the support, are better off under the more unequal distribution G_B. The principal reason for this is that they have lower status in a more equal distribution. See, for example, an agent with income $\hat{z}_-$ in Figure 1, at the lower end of the distribution. Given a fixed income, in a more equal society A, she will occupy a lower social position, as $G_A(\hat{z}_-)$ is less than $G_B(\hat{z}_-)$.

[18] The comparative statics techniques used here are a development of those surveyed in Simon Anderson et al. (2002).

However, the change in the distribution of income will typically also change the equilibrium choice of conspicuous consumption x. As we show later in Proposition 4, depending on the value of minimum status α, conspicuous consumption by the poor may rise or fall when the income distribution becomes more equal. Now, if the choice of x by the poor rises at the same time as their status falls, then clearly the poor are worse off as both status and consumption utility V^* will be lower. Yet, even if the equilibrium level of x were to fall, the resulting increase in consumption utility V^* will not be enough to offset the decrease in status $G(z)$. To see this, note that the Envelope theorem implies that the equilibrium marginal utility of income will be $dU^*/dz = V_2(G(z) + \alpha)$. This is increasing in x by our assumptions on V and clearly also increasing in status $G(z) + \alpha$. So, a lower level of conspicuous consumption and lower equilibrium status both lead to lower marginal utility of income. Equilibrium utility at the lowest income level $\underline{z}$ is fixed by Lemma 1 and will be the same in both cases. Lower marginal utility will therefore lead to lower utility for income levels near $\underline{z}$. Thus, even though conspicuous consumption of the poor is lower, they can be worse off.

Note that even if all the poor behaved as though they were indifferent to status and adopted the cooperative strategy $x_c(z)$, they would still be worse off. This is because $V(x_c, y_c)(G_A(z) + \alpha)$ will be less than $V(x_c, y_c)(G_B(z) + \alpha)$ on the interval $(\underline{z}, \tilde{z})$, simply because there $G_A(z)$ is less than $G_B(z)$. Thus, the poor would be better off in a more unequal society. As the poor have higher utility when there are more poor people around with whom they can make favorable comparisons, we can describe this result as an example of "misery loves company."

We now turn to the effect of an increase in social income in the sense of the first-order stochastic dominance. It turns out that all we have to do is to extend Proposition 2 slightly so that $\tilde{z}$ is now equal to $\bar{z}$, which gives us the following.

COROLLARY 1: *If G_A first order stochastically dominates G_B then for any $\alpha \geq 0$, $U_A^*(z) \leq U_B^*(z)$ for all $z \in [\underline{z}, \bar{z}]$.*

That is, as society becomes more affluent, the above corollary indicates that utility falls at each level of income. This happens for two reasons. First, there is what we could call envy: those with unchanged incomes see their status decreasing as the income of those around rises, because $G_A(z) \leq G_B(z)$ on all of $[\underline{z}, \bar{z}]$. But this is not all. As we will see in Corollary 2 below, in a richer society, expenditure on x typically will be higher. That is, although the individual's own wealth is unchanged, competition for status forces him to increase his expenditure on conspicuous consumption as the incomes of his rivals increase.

We have looked at the effect of changes of the distribution of income on utility at each level of income. Our results make it clear that even an increase in prosperity in the sense of first-order stochastic dominance will have an ambiguous effect on the utilitarian measure of social welfare, that is, average utility. In the more affluent income distribution, G_A, utility is lower at each income level. But equally the fact that G_A first order stochastically dominates G_B means that a positive mass of the population will have higher incomes under G_A than G_B, a rise that may or may not be enough to offset the fall in utility at each income level. Similar issues arise using second-order stochastic dominance, but because of the lack of monotonicity in behavior established in Proposition 2, clear social welfare comparisons are even more difficult. Note, if one looked at changes in utility at a fixed rank or quantile instead of at a fixed income, the results could be more ambiguous.[19] Whether clear results can be obtained on the behavior of utility at fixed ranks rather than at fixed incomes, we leave to further research.

We now turn to comparative statics analysis of the level of conspicuous consumption. We first show that conspicuous consumption is decreasing in the guaranteed minimum level of status α. This implies that conspicuous con-

[19] For example, the type of example exhibited in Figure 1, by Proposition 2, leads to lower utility under distribution G_A for incomes on the range $(\underline{z}, \tilde{z})$. However, by the fact that $G_A(z)$ is lower on that range than $G_B(z)$, an individual occupying the same low rank will have a higher income in society A. Overall, this leads to an ambiguous comparison instead of the clear result of Proposition 2. But equally an individual who has income $\tilde{z}$ has the same position under both distributions but nonetheless will be worse off in the more equal society A.

sumption will be lower for any positive value of α than for α zero. With a minimum level of status guaranteed, competition is muted and conspicuous consumption is reduced.

PROPOSITION 3: *Let $x(z, \alpha^+)$ and $x(z, \alpha^-)$ be the equilibrium conspicuous consumption under two distinct levels of α such that $\alpha^+ > \alpha^- \geq 0$. Then, $x(z, \alpha^+) < x(z, \alpha^-)$ for all $z \in (\underline{z}, \bar{z}]$.*

More surprisingly, perhaps, the guaranteed minimum level of status also affects how equilibrium conspicuous consumption responds to changes in the distribution of income in society. To carry out these comparative statics, we find that first-order stochastic dominance, though a strong condition in itself, is not sufficient in the games of status we consider. Thus we employ the following refinement of first-order stochastic dominance.[20]

Definition (*MLR*).—The two distributions G_A, G_B satisfy the Monotone Likelihood Ratio (MLR) order and we write $G_A \succ_{MLR} G_B$ if the likelihood ratio $L(z) = g_A(z)/g_B(z)$ is strictly increasing on $(\underline{z}, \bar{z}]$.

Just as for our welfare analysis, we are also interested in changes in equality as well as in the wealth of societies. Again, however, second-order stochastic dominance is not sufficient to obtain comparative statics results. Instead we use a strengthening of second-order stochastic dominance analogous to the monotone likelihood ratio order.

Definition (*ULR*).—Two distributions G_A, G_B satisfy the Unimodal Likelihood Ratio (ULR) order and we write $G_A \succ_{ULR} G_B$ if the ratio of their density functions $L(z) = g_A(z)/g_B(z)$ is unimodal and $\mu_A \geq \mu_B$. That is, L is strictly increasing for $z < \hat{z}$ and it is strictly decreasing for $z > \hat{z}$ for some $\hat{z} \in (\underline{z}, \bar{z}]$.

In simple terms, if an income distribution $G_A \succ_{ULR} G_B$, then G_A is more equal and less dispersed than G_B. More precisely, it can be shown that if $G_A \succ_{ULR} G_B$, then G_A also second order stochastically (equivalently, generalized Lorenz) dominates G_B.[21] This in turn implies that if $G_A \succ_{ULR} G_B$ and the means are in fact equal, then G_B is a mean-preserving spread of G_A. An example of two distributions that satisfy this relationship is given in Figure 1. If $G_A \succ_{ULR} G_B$ then the ratio $L(z)$ will have a unique maximum on $(\underline{z}, \bar{z}]$.[22] Let $\hat{z}$ be that value of z that maximizes the ratio. If $\hat{z} = \bar{z}$, then the above condition reduces to the monotone likelihood ratio order, or in other words, the monotone likelihood ratio order implies the unimodal likelihood ratio order.

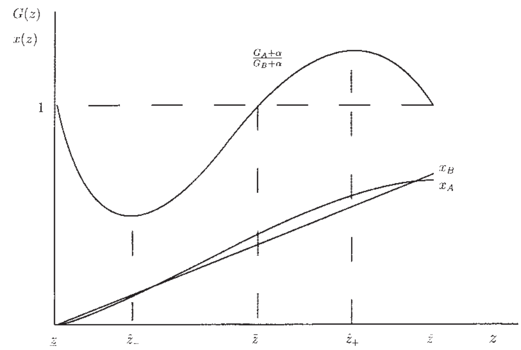

FIGURE 2. COMPARATIVE STATICS OF SOLUTIONS WITH AN INCREASE IN EQUALITY

We are able to obtain comparative statics results on solutions to the differential equation (6) by making use of the ratio $P(z, \alpha) = (\alpha + G_A(z))/(\alpha + G_B(z))$. As Lemma A2 in the Appendix shows, if the distribution G_A dominates G_B in the sense of the ULR order then the ratio $(\alpha + G_A(z))/(\alpha + G_B(z))$ has at most two extremal points, $\hat{z}_-$, where the ratio is at minimum, and $\hat{z}_+$, where the ratio is at maximum, with $\underline{z} \leq \hat{z}_- < \tilde{z} < \hat{z} < \hat{z}_+ < \bar{z}$.[23] An example of this is illustrated in Figure 2. Together with $\tilde{z}$, which will be the crossing point of the two

[20] Researchers have resorted to similar refinements to obtain monotone comparative statics in games of incomplete information, such as auctions. For a concise survey of many relevant results, see Appendix B of Vijay Krishna (2002).

[21] The ULR order and this further result were introduced by Hector M. Ramos et al. (2000) in the context of the measurement of inequality. The properties of this and other ratio orders and their implications for comparative statics are explored further in Hopkins and Kornienko (2003).

[22] The condition that $\mu_A \geq \mu_B$ rules out the possibility that the maximum is, for example, at the lower bound $\underline{z}$ which would imply that $G_B \succ_{MLR} G_A$.

[23] Note that $\underline{z} = \hat{z}_-$ if and only if $\alpha = 0$. That is, $P(z, 0)$ is increasing on $(\underline{z}, \tilde{z})$.

distribution functions, these two extremal points provide a convenient interpretation. If individual's income z is in the interval $[\underline{z}, \hat{z}_-]$, we say that this individual is "poor;" if $z \in (\hat{z}_-, \hat{z}_+)$, she belongs to the "middle class;" if $z \in (\hat{z}_+, \bar{z}]$, she is "rich." We are now able to state our main comparative static result on conspicuous consumption.[24]

PROPOSITION 4: *Suppose $x_A(z)$ and $x_B(z)$ are the equilibrium choices of the positional good for distributions G_A and G_B, respectively. If $G_A(z) \succ_{ULR} G_B(z)$, then*

(*i*) *if $\alpha = 0$, $x_A(z)$ crosses $x_B(z)$ at most once. Moreover, $x_A(z) > x_B(z)$ for all $z \in (\underline{z}, \hat{z}_+]$, with a possible crossing on $(\hat{z}_+, \bar{z}]$;*
(*ii*) *if $\alpha > 0$, $x_A(z)$ crosses $x_B(z)$ at most twice. Moreover, $x_A(z) < x_B(z)$ for all $z \in (\underline{z}, \hat{z}_-]$, with a crossing in $(\hat{z}_-, \tilde{z})$ so that $x_A(z) > x_B(z)$ for all $z \in [\tilde{z}, \hat{z}_+]$, with a possible crossing on $(\hat{z}_+, \bar{z}]$.*

Let us go through this result in less formal terms. An increase in equality in terms of the ULR order will typically mean a greater density of people with middle incomes (see for example, Figure 1, where $g_A(z) > g_B(z)$ in the central section of the support of the two distributions). The intuitive effect of this is clear: the closer together are people, the greater the incentive to differentiate oneself, that is, in a more densely packed society the marginal return to conspicuous consumption is higher. Indeed the above result indicates that the conspicuous consumption of the middle class rises.

If the distribution G_B is a mean-preserving spread of G_A, then in the more equal society A, the population density will typically be thinner in the tails of the distribution. Social competition will tend to be reduced at high and low incomes. The conspicuous consumption of the rich, that is, agents with incomes more than $\hat{z}_+$, consequently may fall. This is illustrated in Figure 2, which shows sample solutions for two income distributions where G_B is a mean-preserving spread of G_A. At income levels above $\hat{z}_+$, the two solutions may cross once as depicted in Figure 2. But equally for some pairs of distributions the conspicuous consumption of the rich will be unambiguously higher in the more equal society.

The effect on the poor is somewhat different. Population density at low income levels will be lower which would tend to reduce competitive pressure. However, at low incomes, status is now lower. For example, in Figure 1, $G_A(z) < G_B(z)$ for all income levels below the median income $\tilde{z}$: these people have been left behind. Their response differs, depending on whether the minimum level of status α is positive. With α positive, Proposition 4 indicates the conspicuous consumption of the poor, that is, those with income less than or equal to $\hat{z}_-$, will fall as the income distribution becomes more equal. But in contrast, when $\alpha = 0$, the consequence of low status is serious and all are desperate to avoid it. Consequently, we find that after a rise in equality those who still have low incomes will raise their conspicuous consumption in order to keep up.

We are also interested in what happens to the demand for positional goods as a society's income increases. As stated above the MLR order implies the ULR order. Hence, we can derive the following corollary as the special case of the above proposition when $\tilde{z} = \bar{z}$.

COROLLARY 2: *Suppose $x_A(z)$ and $x_B(z)$ are the equilibrium choices of the positional good for distributions G_A and G_B respectively. If $G_A(z) \succ_{MLR} G_B(z)$, then*

(*i*) *if $\alpha = 0$, $x_A(z) > x_B(z)$ for all $z \in (\underline{z}, \bar{z})$;*
(*ii*) *if $\alpha > 0$, $x_A(z)$ crosses $x_B(z)$ once. Moreover, $x_A(z) < x_B(z)$ for all $z \in (\underline{z}, \hat{z}_-]$, then crosses in $(\hat{z}_-, \bar{z})$ so that $x_A(\bar{z}) > x_B(\bar{z})$.*

That is, an increase in a society's average income can have two quite different results. Specifically, when individuals are "desperate" ($\alpha = 0$), such an increase leads to a general increase in expenditure on conspicuous consumption and expenditure rises at every level of

[24] Strictly speaking, the inequalities of Proposition 4 and subsequent results should be qualified as holding "almost everywhere" as we cannot absolutely rule out points of tangency between $x_A(z)$ and $x_B(z)$. Note, however, that such tangency is nongeneric as it can happen only at the inflection points of $P(z, \alpha)$. In turn, the inflection points in $P(z, \alpha)$, if any exist, can happen only at the turning point $\hat{x}$ and inflection points of $L(z)$. Since this does not affect the qualitative nature of our results, we avoid repetition of this technical qualification.

income. In contrast, if guaranteed minimum status α is positive, the poorest members of society actually spend less, and the rich and, possibly, some of the middle classes increase expenditure. In either case, as we have seen by Corollary 1, utility will fall at every level of income. This offers a potential explanation for the phenomenon, first identified by Easterlin (1974), that happiness scores in the developed world do not seem to have been rising over time, despite substantial increases in average income.

III. Consumption Taxes and Subsidies

As we have seen above in Section I, conspicuous consumption is "wasteful" in the sense that it leads to a Pareto-inferior outcome. Frank (1985b), who was the first to produce a result of this kind, compares conspicuous consumption to pollution, in that it imposes a negative externality on other consumers (Frank, 1999). As a potential correction for the externality, Frank (1985a, b, 1999) has advocated a consumption tax, in effect a Pigouvian tax. Many governments in the past have labelled certain products as luxuries and levied taxes on them. However, what is and what is not a luxury is somewhat subjective and taxes imposed on this basis seem likely to produce unwanted distortions. Frank has suggested instead that the tax fall on total consumption (this gives the nonconspicuous good y an attractive interpretation as saving). We try to identify a tax policy that could implement the cooperative solution identified in Section I, that if achieved, would represent a Pareto improvement on the Nash equilibrium. However, it is not an optimal tax in the conventional sense in that it does not attempt to maximize a social welfare function. We discuss the implications of this approach at the end of this section.

A Pigouvian tax on an externality involves raising the price of the externality-causing good or activity until it reaches its social cost. In the present model, the return to additional expenditure on x is typically different at different levels of income. Therefore, in order for the government to implement the cooperative solution it may have to use what amounts to perfect price discrimination, charging a different level of consumption tax and/or offering a different level of subsidy at each level of income. Denote the posttax relative price as $p_\tau(z) = p(1 + \tau(z))$ where p is the initial relative price. In particular, suppose there is a policy $\tau(z)$ such that in Nash equilibrium the cooperative solution (7) is chosen, then one can write the differential equation (6) that defines the equilibrium as

$$(11)\quad x'(z, p_\tau) = \left(\frac{g(z)}{G(z) + \alpha}\right)\left(\frac{V(x_c, y_c)}{p_\tau V_2(x_c, y_c) - V_1(x_c, y_c)}\right) = \left(\frac{g(z)}{G(z) + \alpha}\right)\left(\frac{V(x_c, y_c)}{p\tau(z)V_2(x_c, y_c)}\right).$$

And, if the solution of this equation is indeed equal to the cooperative solution, then at each z, it must hold that $x'(z, p_\tau) = x_c'(z, p)$. Substituting this into the above equation and solving for $\tau(z)$, one obtains,

$$(12)\quad \tau(z) = \left(\frac{g(z)}{G(z) + \alpha}\right)\left(\frac{V(x_c, y_c)}{pV_2(x_c, y_c)x_c'(z, p)}\right).$$

Thus, there exists a continuous function $\tau(z)$ that, if $p_\tau(z) = p(1 + \tau(z))$, implements the cooperative solution. That is, $x(z, p_\tau) = x_c(z, p)$ and $y(z, p_\tau) = y_c(z, p)$ at each income level z.

Note that there are several ways in which the appropriate relative price can be obtained. One would be simply to tax x. However, since our model for the present lacks a public sector, we concentrate on revenue-neutral policies. As Ireland (1994) proposes, the revenue raised could be used to subsidize the price of nonconspicuous consumption. We give an example of this.

Example 1: Suppose that $U = xyS(x, F(\cdot))$ and that z is distributed uniformly on [0, 1], then in the equilibrium each consumer demands $2z/(3p)$ units of x and $z/3$ units of y. In this case the "cooperative" allocation is $x_c(z) = z/(2p)$ and $y_c(z) = z/2$. From the equation (12), one can calculate that in this case $\tau(z) = 1$, a constant. If initially, $p = 1$, then one policy that would implement the cooperative case and which would be revenue neutral would be to tax x at a tax rate of ⅓ and subsidize y at a subsidy rate of ⅓.

More generally, a tax policy derived in this way is unlikely to be constant. This raises the question as to whether policy intervention should be greatest at low or high incomes. What we can show is that the comparative statics techniques developed in the previous section can also be fruitfully applied in this context.

PROPOSITION 5: *Suppose there are two income distributions G_A, G_B such that $G_A \succ_{ULR} G_B$. Let $\hat{z}_-$, $\hat{z}_+$ be respectively the minimum and the maximum of the ratio $(G_A(z) + \alpha)/(G_B(z) + \alpha)$. Further, let the two resulting consumption taxes, as defined by (12), under the two different distributions be $\tau_A(z)$ and $\tau_B(z)$ respectively. Then, $\tau_A(z) < \tau_B(z)$ on $(\underline{z}, \hat{z}_-)$, $\tau_A(z) > \tau_B(z)$ on $(\hat{z}_-, \hat{z}_+)$ and $\tau_A(z) < \tau_B(z)$ on $(\hat{z}_+, \bar{z}]$.*

That is, with an increase in equality, taxes (and subsidies) should definitely increase for the "middle class" but possibly decrease for the poor and for the rich. The intuition for this result is that for income ranges where conspicuous consumption rises, intervention should rise in those areas also. We have already seen in Section II that an increase in equality in terms of the unimodal likelihood ratio order will lead to an increase in conspicuous consumption by the middle classes and possibly a decrease by the rich. This fact is reflected in the above proposition which establishes that if there is an increase in equality, the marginal tax and subsidy rate should be increased for those of middle income and decreased for those with high income. Again, what happens to the poor depends on the guaranteed minimum level of status α. If α is positive, then the taxes and subsidies at low levels of income should fall. If α is zero, then $\hat{z}_-$ is in effect equal to $\underline{z}$, and taxes and subsidies for the poor should rise.

It might have seemed a reasonable hypothesis that policy intervention should be largest at high incomes. As the rich spend more on luxuries, the most conspicuous form of conspicuous consumption, it might seem that this form of expenditure should be the most taxed. However, this is not what we find. Rather, the formula for the corrective tax emphasizes that intervention should be the greatest where the degree of social competition is highest and this is likely to be near the modal level of income.[25] Though, in the current framework it might not be appropriate to call such a policy regressive, as whoever has higher taxes also receives greater subsidies. Now, we have seen in Example 1 above that, for Cobb-Douglas type preferences and a uniform distribution of income, the consumption tax is constant. Thus, for these preferences and for any distribution which dominates the uniform in the sense of the ULR order, the corrective policy will have a higher marginal rate of tax and subsidy for the poor than the rich. Note that any distribution that has a mean that is no lower and a unimodal density will dominate the uniform in the sense of the ULR order. Thus, for any distributions that resemble actual empirical income distributions, and for preferences as in Example 1, this analysis suggests that intervention should be larger at low income levels.[26] This may be surprising, but note that such a tax, combined with appropriate subsidies, will implement the cooperative solution and make everybody better off. We now offer a simple example of this.

Example 2: If $G_A(z) = 3z^2 - 2z^3$ (a unimodal Beta distribution) on [0, 1] then if $G_B(z) = z$ (a uniform distribution) then G_B is a mean-preserving spread of G_A and $G_A \succ_{ULR} G_B$. For the preferences $U = xyS(x, F(\cdot))$, one can calculate that $\tau_A(z) = 6(1 - z)/(3 - 2z)$. This can be implemented revenue neutrally by a tax on x equal to $\tau_x(z) = 3(1 - z)/(6 - 5z)$ and an equal subsidy on y. The function τ_x is strictly decreasing with $\tau_x(0) = 0.5$ and $\tau_x(1) = 0$. That is, the poorest face a marginal tax rate of 50 percent and the rich a zero marginal rate.

Earlier work on this topic was done by George Kosicki (1989). In a nonstrategic setting, similar to that of Frank (1985b), in which

[25] If the distribution of income is unimodal, the ratio $g(z)/(G(z) + \alpha)$, on which the expression (12) depends, will have its maximum at less than the modal level of income.

[26] The preferences in this example implicitly assume that $\alpha = 0$. Proposition 5 implies that for a positive α the appropriate tax/subsidy for an income distribution more equal than the uniform would be higher for the middle classes than for the poor or rich. So, it is not strictly decreasing, but it is still regressive in the sense that in the optimal tax literature, progressivity is associated with a convex tax schedule. See, for example, Ireland (2001).

agents care about status, he found that the optimal income tax could be regressive. Ireland (1994, 2001) has also investigated the possibility of Pareto-improving taxes in the presence of concern for status. He analyzes models where agents of different abilities signal their absolute type through wasteful consumption. He finds that taxes on signaling do increase welfare. In his later work, he reports status concerns give little support for progressive taxation, even if they do not support regressive taxation.

Note, however, the important differences between our approach and that of Ireland (1994, 2001). First, in the models he considers, payoffs depend on perception of absolute type, rather than relative position, as payoffs do here. Second, our tax policy only corrects the consumption externality arising from concern for relative position. Tax policy in Ireland (1994) was treated mostly in terms of some examples, whereas Ireland (2001) goes on to derive the tax that maximizes welfare. While the latter approach is more complete, it combines two different ways in which taxes in such a model can be welfare improving. Moreover, these can pull in different directions. First, we have seen that taxes that correct the externality may be highest at low incomes. Second, once this correction has taken place, if the utility function $V(x, y)$ were sufficiently concave, one could then increase a utilitarian measure of welfare (that is, average utility) further by progressive taxes and redistribution for the standard reason that with concave utility, the marginal utility of income is decreasing. These two opposing effects may be why Ireland's (2001) results are relatively ambiguous.

IV. Conclusion

We considered a game of status, in which individuals are concerned with the level of their consumption relative to that of others as well as its absolute level. Individual status is determined by the consumption choices of others in the equilibrium of the resulting game. In the symmetric Nash equilibrium, an individual's status, that is, her position in the distribution of consumption, coincides with her position in the income distribution. Hence, rank in the income distribution can be inferred from one's consumption behavior. But this signalling is costly and the Nash equilibrium is Pareto dominated by the state where agents take no account of status in their consumption decisions.

The advantage of a simultaneous move game approach is that it is possible to analyze how exogenous changes in the distribution of income affect individual decisions. We show that, for the class of preferences considered, an increase in average income, in the sense of a refinement of first-order stochastic dominance, will tend to lead to an increase in conspicuous consumption. Furthermore, a more affluent society will have lower utility at each income level. Hence, an increase in average income may be consistent with a decrease in social welfare. More plausibly, social welfare may rise only slowly in response to economic growth. This offers one explanation for the empirical finding, first due to Easterlin (1974), that average happiness scores seem to change more slowly than average income.

We also analyze the effect of changes in inequality on conspicuous consumption and welfare. We find that the poor are made worse off by greater equality. Such an increase in equality will also increase conspicuous waste by those on middle incomes or low and middle incomes, while conspicuous consumption by the rich may decrease. Consequently, when we consider corrective taxes on conspicuous consumption, we find that the more equally income is distributed, the higher taxes and subsidies should be for those with low to middle incomes, and the lower they should be for those with high incomes. Indeed, we are able to show that in some circumstances a corrective policy with higher tax rates (but also higher subsidies) for those with low incomes can lead to a Pareto improvement.

These results may seem counterintuitive, and indeed it is important to interpret them with care. First, the strength of our results is based on the comparison of the effects of different income distributions on individuals at a given income level. For some changes in income distribution, for example those that move everyone forward, an alternative might be to examine the effect at a given rank in the income distribution. This is less easy to do and we leave such analysis to future research. Second, our results do not imply, for example, that making the current tax structure more regressive would necessarily increase social welfare. Rather, the present work investigates the logical consistency of the hypothesis that a concern for status would make greater equality more desirable. In particular, relative concerns have been advanced recently as an additional and compelling reason for progressive taxation. Our results cast doubt on this argument and suggest that the presence of relative concerns does not provide an additional rationale for progressive taxation over and above the oldest one: that an additional dollar is likely to be worth less to a millionaire than to a pauper.

A related question is whether a different model of relative concerns could allow for greater equality to have an unambiguously positive impact on welfare. It is true that the sharpness of our results does depend on the particular form of preferences that we employ. Nonetheless, our results still hold even when the poor are not deeply concerned about status. Meanwhile, Larry Samuelson (2004) investigates a model where agents are concerned about the consumption decisions of others not for reasons of status but because consumption patterns provide information about economic opportunities. In such a society an increase in equality, in that it increases the precision of information on the underlying environment, may be welfare improving. This difference in outcome is largely due to the fact that, in the model of Samuelson (2004), an individual's consumption choices do not impose direct negative externalities on others.

This is important because, as our analysis indicates, the exact relation between inequality and consumption behavior has important implications. It seems to imply that policy interventions to reduce inequality might have unintended consequences. Furthermore, while in the framework we consider here a consumption tax, combined with an appropriate subsidy, will be welfare improving, the form it should take depends on the distribution of income in unexpected ways. These findings should not be interpreted as a call for the immediate overhaul of existing attitudes and policies toward inequality. Our current understanding of relative concerns, their place in human happiness and the interaction with income inequality is much too rudimentary for that. However, we think that our present work has illuminated some connections between status and inequality which are both complex and unexpected. We hope that we have shown that this interaction is worthy of further study and that the present work provides a methodology for its analysis.

Appendix

PROOF OF LEMMA 1:

In a symmetric equilibrium, an individual with income $\underline{z}$ has rank 0.

(i) If the parameter α is zero, then her equilibrium utility is $U^*(\underline{z}) = V(x, y)G(\underline{z}) = 0$. The only way she can increase her utility would be to raise her rank. Thus, in equilibrium the expenditure of those with slightly greater income must be such that the poorest agent is unable to increase her rank by increasing expenditure on x. That is, $\lim_{z \to \underline{z}^+} x(z) \geq \underline{z}/p$. But, as necessarily, $x(z) \leq z/p$ for all z, we have $\lim_{z \to \underline{z}^+} x(z) = \underline{z}/p$. Consequently, $\bar{S}(x, F(\cdot)) = 0$ for any $x \leq \underline{z}/p$, and so agents with income $\underline{z}$ are indifferent between any level of x between 0 and $\underline{z}/p$.

(ii) Alternatively, suppose α is positive. Then $U^*(\underline{z}) = V(x, y)\alpha$ and does not depend on the agent's rank. Therefore, in equilibrium her choice must maximize $V(x, y)$. That is, she must choose $x_c(\underline{z})$, or there would be a profitable deviation.

PROOF OF PROPOSITION 1:

The proof has the following structure. We first establish that an equilibrium strategy will be increasing, then continuous, then differentiable. It will then be characterized by the differential equation (6), for which a solution must exist and that will be unique on the interval $(\underline{z}, \bar{z}]$.

LEMMA A1: *If the strategy $x^*(z)$ is a best response to other agents' consumption choices then it is increasing.*

PROOF:

This result is adapted from Proposition 1 of Maskin and Riley (2000) for high bid auctions. If an agent i with income z_i adopts the choice $x_i = x^*(z_i)$ which is a best response to choices of other agents summarized by the distribution $F(\cdot)$, then we have necessarily $x_i \geq x_c(z_i)$, the cooperative level defined in (7), as it is strictly dominated to choose a level of consumption below the cooperative level. Suppose that equality holds, that is, the best reply for an agent with income z_i is to choose $x_c(z_i)$. But one can verify that $x_c(z)$ is increasing by the assumptions that $V_{ii} \leq 0$ and $V_{ij} \geq 0$.

We now turn to the case of best responses where $x_i > x_c(z_i)$. For any other choice, $\check{x}_i \in (x_c(z_i), x_i)$, necessarily

$$V(x_i, z_i - px_i)S(x_i, F(\cdot)) \geq V(\check{x}_i, z_i - p\check{x}_i)S(\check{x}_i, F(\cdot)). \tag{A1}$$

The next step is to establish the following inequality:

$$\frac{\partial V}{\partial z_i}(x_i, z_i - px_i)S(x_i, F(\cdot)) > \frac{\partial V}{\partial z_i}(\check{x}_i, z_i - p\check{x}_i)S(\check{x}_i, F(\cdot)). \tag{A2}$$

One can write the left-hand side of the above as

$$\frac{\partial V}{\partial z_i}(x_i, z_i - px_i)S(\check{x}_i, F(\cdot)) + \frac{\partial V}{\partial z_i}(x_i, z_i - px_i)(S(x_i, F(\cdot)) - S(\check{x}_i, F(\cdot))). \tag{A3}$$

Now, the first term in the above must be at least as big as the right-hand side of (A2) by our assumptions that $V_{ij} \geq 0$ and that $V_{ii} \leq 0$, which ensures that $\partial V/\partial z$ is increasing in x. Now, if x_i is greater than $x_c(z_i)$, $V(x_i, z_i - px_i)$ is decreasing in x_i. Hence, if x_i is a best response then $S(x_i, F(\cdot)) > S(\check{x}_i, F(\cdot))$ or the agent i could lower x_i to $\check{x}_i$ with no loss of status and an increase in utility. Then the second term in the above equation must be strictly positive as V_2 is strictly positive by assumption. We have established the inequality (A2), so that at x_i, $\partial U/\partial z$ is strictly increasing in x. But equally this implies an increase in income leads to an increase in the marginal return to x, and the optimal choice of x necessarily increases.

We now turn to the characterization of the symmetric equilibrium strategy $x(z)$. From Lemma A1, $x(z)$ is an increasing function of an agent's income. Suppose that it is not strictly increasing so that $\check{x} = x(z_0) = x(z_1)$ for some $z_0 < z_1$. Then, $F(\check{x}) > F^-(\check{x})$, that is, there is a mass point in the distribution of consumption at $\check{x}$. But then $S(\check{x}, F(\cdot)) < F(\check{x}) \leq S(\check{x} + \varepsilon, F(\cdot))$ for all $\varepsilon > 0$. That is, any small increase in x will lead to a discrete increase in status. But V is continuous in x, so there must exist an increase in x sufficiently small that the consequent increase in status will be greater than any decrease in direct utility V. That is, there is a profitable deviation, which must be feasible for an agent with income z_1 as $z_1 > z_0 \geq p\check{x}$. Hence, a symmetric equilibrium strategy must be strictly increasing.

Furthermore, in a symmetric equilibrium the equilibrium strategy $x(z)$ will be continuous. Suppose not, so there is a jump upwards in the equilibrium strategy at some income level $\check{z}$ so that $\lim_{z \to \check{z}} x(z) = \hat{x} \neq x(\check{z})$. Note, that as $x(z)$ is strictly increasing, despite the discontinuity at $\check{z}$ we have $\lim_{x \to x(\check{z})} S(x, F(\cdot)) = S(x(\check{z}), F(\cdot)) = S(\hat{x}, F(\cdot))$. An individual with income $\check{z} + \varepsilon$ for $\varepsilon > 0$ who decreases her consumption from $x(\check{z} + \varepsilon)$ to $\min[\hat{x}, x(\check{z})]$, that is to the level at the bottom of the jump, will gain a discrete increase in direct utility V. But by the continuity of S on $[x(\check{z}), x(\check{z} + \varepsilon)]$, there must exist an ε sufficiently small that the consequent decrease in status will be smaller than the increase in direct utility V. That is, there is a profitable deviation.

So, the equilibrium strategy is continuous and strictly increasing and therefore the inverse function

$x^{-1}(\cdot)$ is well defined. It follows then that the probability that an individual i has higher status than another individual j is

$$F(x_i(z_i)) = \Pr[x_i(z_i) > x_j(z_j)] = \Pr[x_j^{-1}(x_i(z_i)) > z_j] = G(x_j^{-1}(x_i(z_i))) = G(z_i) \tag{A4}$$

The final step follows from the assumption of a symmetric equilibrium, where $x_i(z_i) = x_j(z_i)$. Hence, $x_j^{-1}(x_i(z_i)) = z_i$.

Finally, the differentiability of the equilibrium function $x(z)$ on $(\underline{z}, \bar{z}]$ can be established using standard arguments from auction theory (here we follow Maskin and Riley, 1984, pp. 1485–86). Note that if $\check{z} = z + \Delta z$, then for any Δz

$$V(x(z), z - px(z))(\alpha + G(z)) \geq V(x(\check{z}), z - px(\check{z}))(\alpha + G(\check{z})).$$

That is, for an agent with income z choosing x according to the equilibrium strategy $x(z)$ must give at least as much utility as choosing an alternative amount $x(\check{z})$. Similarly, for an agent with income $\check{z}$ choosing x according to the equilibrium strategy $x(\check{z})$ must give at least as much utility as choosing an alternative amount $x(z)$, i.e.

$$V(x(\check{z}), \check{z} - px(\check{z}))(\alpha + G(\check{z})) \geq V(x(z), \check{z} - px(z))(\alpha + G(z)).$$

By the mean value theorem, we have

$$V(x(z), z - px(z))(G(z) - G(\check{z}))$$
$$- (V_1(x_1, z - px_1) - pV_2(x_1, z - px_1))(\alpha + G(\check{z}))(x(\check{z}) - x(z)) \geq 0$$

and

$$V(x(\check{z}), z - px(\check{z}))(G(\check{z}) - G(z))$$
$$- (V_1(x_2, \check{z} - px_2) - pV_2(x_2, \check{z} - px_2))(\alpha + G(z))(x(z) - x(\check{z})) \geq 0$$

where x_1, x_2 are both between $x(z)$ and $x(\check{z})$. Rearranging, dividing by Δz and combining the above two inequalities, we obtain the following double inequality:

$$\frac{V(x(\check{z}), \check{z} - px(\check{z}))(G(\check{z}) - G(z))}{(pV_2(x_2, \check{z} - px_2) - V_1(x_2, \check{z} - px_2)(\alpha + G(z))\Delta z} \geq \frac{x(\check{z}) - x(z)}{\Delta z}$$
$$\geq \frac{V(x(z), z - px(z))(G(\check{z}) - G(z))}{(pV_2(x_1, z - px_1) - V_1(x_1, z - px_1))(\alpha + G(\check{z}))\Delta z}$$

But as $x(\cdot)$ is continuous, the left- and right-most terms of this double inequality converge to the right-hand side of (6) as Δ goes to zero. Thus, $x(z)$ is differentiable and thus satisfies (6) on $(\underline{z}, \bar{z}]$.

We now check that the first-order conditions that define the differential equation (6) actually represent optimal behavior for each agent. A sufficient condition is pseudoconcavity. That is, U is increasing in x for $x < x(z)$ and decreasing for $x > x(z)$. Now, take $\check{x} < x(z)$ and let $\check{z}$ be such that $x(\check{z}) = \check{x}$. Then, $\check{z} < z$. Conditional on $dU/dx = 0$ and all other agents adopting the equilibrium symmetric strategy we have $\partial^2 U/\partial x \partial z = (V_{12} - pV_{22})(\alpha + F(x)) + V_2 f(x) > 0$ almost everywhere. Hence, for any $\check{x} < x(z)$, $dU(\check{x}, z)/dx \geq dU(\check{x}, \check{z})/dx = 0$. This shows that U is increasing in x for $x < x(z)$. One can similarly show that it is decreasing for $x > x(z)$.

Any symmetric equilibrium strategy $x(x)$ will exceed the cooperative equivalent $x_c(z)$ except perhaps at $\underline{z}$. Suppose not, so that a symmetric equilibrium strategy, $x(\check{z}) = x_c(\check{z})$ for some $\check{z} \in (\underline{z}, \bar{z}]$, then at that point $V_1(x(\check{z}), \check{z} - px(\check{z}) - pV_2(x(\check{z}), \check{z} - px(\check{z})) = 0$ so that the left-hand side of the first-order conditions (5) will be positive. That is, an agent choosing the cooperative level would in fact have an incentive to increase her conspicuous consumption x.

We are left with establishing that the differential equation arising from the first-order conditions (5) describes an essentially unique symmetric equilibrium. If $\alpha > 0$, this just follows from the fundamental theorem of differential equations [if $x(z)$ is continuously differentiable, then so is (6) and so it has a unique solution]. However, in the case where $\alpha = 0$, if we evaluate $x'(z)$ at $z = \underline{z}$, we have the denominator equal to zero and there is a potential failure of Lipschitz continuity (a problem known from the analysis of auctions), and therefore there are potentially multiple solutions with the same boundary condition. We can rule this out here. Note that as we have established $x(z)$ is continuously differentiable on $(\underline{z}, \bar{z}]$, the differential equation (6) has a unique solution on $(\underline{z}, \bar{z}]$. Thus, any potential multiple solutions cannot cross on $(\underline{z}, \bar{z}]$. But then any two solutions, say $x_1(z)$, $x_2(z)$ must satisfy $x_2(z) > x_1(z)$ for $z > \underline{z}$. Note, first, that for $\psi(x, z) = V/(pV_2 - V_1)$, given our assumptions on the second derivatives of V, we have

$$\frac{\partial \psi}{\partial x} = \frac{-(pV_2 - V_1)^2 - V(-p^2V_{22} + pV_{21} - V_{11} + pV_{12})}{(pV_2 - V_1)^2} < 0 \tag{A5}$$

Examining the equation (6) we have $x_2'(z) < x_1'(z)$ for $z > \underline{z}$. This would imply $\int_{\underline{z}}^{\bar{z}} x_1'(z)\,dz > \int_{\underline{z}}^{\bar{z}} x_2'(z)\,dz$. Now, as $\lim_{z \to \underline{z}^+} x_1(z) = \lim_{z \to \underline{z}^+} x_2(\underline{z})$, this would in turn imply that $x_1(\bar{z}) > x_2(\bar{z})$ which is a contradiction. Thus, both for α zero and α positive, there is exactly one solution that satisfies the boundary conditions given in Lemma 1 and that will be the unique symmetric equilibrium.

PROOF OF PROPOSITION 2:

We have assumed that G_A and G_B are equal at a finite number of points and so the number of crossing points is also finite.[27] Let $\tilde{z}$ be the first crossing point in the interval $(\underline{z}, \bar{z})$ (and if there are no crossing points let $\tilde{z}$ be equal to $\bar{z}$). If condition (10) holds, it clearly cannot be the case that $G_A(z) > G_B(z)$ on $(\underline{z}, \tilde{z})$. Hence, $G_A(z) \le G_B(z)$ on $(\underline{z}, \tilde{z})$.

Next, note that the function $U^*(z)$ is continuously differentiable as $x(z)$ and $G(z)$ are continuously differentiable. We now prove the following claim: if $G_A(z) \le G_B(z)$ for all $z \in [\underline{z}, \tilde{z}]$ (where $\tilde{z} > \underline{z}$ corresponds to the first crossing of $G_A(z)$ and $G_B(z)$), then $U_A^* \le U_B^*$ for all $z \in [\underline{z}, \tilde{z}]$. Given the common boundary conditions established by Lemma 1, for $\alpha > 0$, we have $U_A^*(\underline{z}) = U_B^*(\underline{z}) = U_c(\underline{z})$ and for $\alpha = 0$, we have $U_A^*(\underline{z}) = U_B^*(\underline{z}) = 0$. The Envelope theorem implies that $dU^*(z)/dz = U^{*\prime} = V_2^*(\alpha + G(z))$. As Lemma A1 established the inequality (A2), we have $dU^*(z)/dz$ increasing in x. In equilibrium, $x^*(z) > x_c(z)$ (except perhaps at $\underline{z}$). It follows by the strict quasiconcavity of V that $V_1(x^*, z - px^*) - pV_2(x^*, z - px^*)$ is negative and so, in the neighborhood of x^*, $V(x, z - px)$ is strictly decreasing in x.

Suppose the claim is false, and there exists at least one interval on $(\underline{z}, \tilde{z}]$ where $U_A^*(z) > U_B^*(z)$. Let us denote the set of points as $I_U = \{z \le \tilde{z} : U_A^*(z) > U_B^*(z)\}$ (possibly disjoint), and let $z_1 = \inf I_U \ge \underline{z}$. We can find a $z_2 \in I_U$ such that $U_A^*(z) > U_B^*(z)$ for all z in $(z_1, z_2]$. Note that since, by the common boundary condition, $U_A^*(\underline{z}) = U_B^*(\underline{z})$, we can rule out the case where $U_A^*(z_1) > U_B^*(z_1)$, so that $U_A^*(z_1) = U_B^*(z_1)$.[28] As $U_A^*(z) > U_B^*(z)$ and $G_A(z) \le G_B(z)$ for all $z \in I_U$, it must be that $V(x_A(z), z - px_A(z)) > V(x_B(z), z - px_B(z))$ and hence $x_A(z) < x_B(z)$ for all $z \in I_U$. But then as $U^{*\prime}$ is increasing in $x(z)$ and strictly increasing in $G(z)$, we have $U_A^{*\prime}(z) \le U_B^{*\prime}(z)$ on I_U. This, together with $U_A^*(z_1) = U_B^*(z_1)$, implies $U_A^*(z) \le U_B^*(z)$ for all $z \in (z_1, z_2]$, which is a contradiction.

[27] This is to avoid certain pathological examples where the function $G_B(z) - G_A(z)$ changes sign infinitely often (so there is no "first" crossing point) and yet the condition (10) still holds. An alternative way to avoid this problem would be to assume that $g_A(\underline{z}) < g_B(\underline{z})$, a condition that would hold if, for example, $G_A \succ_{ULR} G_B$.

[28] For example, it is not possible that $U_A^*(z) > U_B^*(z)$ on $[\underline{z}, z_2]$.

PROOF OF PROPOSITION 3:

Notice first that $g(z)/(G(z) + \alpha^-) > g(z)/(G(z) + \alpha^+)$ for all z. If $x(z, \alpha^-)$ and $x(z, \alpha^+)$ cross at some point $\check{z}$ then, examining the differential equation (6), it is clear that $x'(\check{z}, \alpha^-) > x'(\check{z}, \alpha^+)$. That is, $x(z, \alpha^+)$ crosses $x(z, \alpha^-)$ from above and there can be at most one such crossing. Suppose there a crossing. Then, if $x(\underline{z}, \alpha^+) < x(\underline{z}, \alpha^-)$ (which is possible only if $\alpha^- = 0$), we have a contradiction, and the proof is complete. If $x(\underline{z}, \alpha^+) = x(\underline{z}, \alpha^-)$ (which by Lemma 1 will certainly be the case if both α^- and α^+ are both strictly positive), then there must be a point $\dot{z}$ in $(\underline{z}, \check{z})$ where the difference $x(z, \alpha^+) - x(z, \alpha^-)$ is maximized and so that $x'(\dot{z}, \alpha^+) = x'(\dot{z}, \alpha^-)$. But given that by (A5), $\psi(z, x)$ is decreasing in x, this generates a contradiction.

PROOF OF PROPOSITION 4:

Given that $P(z, \alpha) = (\alpha + G_A(z))/(\alpha + G_B(z))$, then the inequality $\partial P(z, \alpha)/\partial z > 0$ is equivalent to the following two conditions

$$\text{(A6)} \qquad \frac{g_A(z)}{G_A(z) + \alpha} > \frac{g_B(z)}{G_B(z) + \alpha} \Leftrightarrow L(z) > P(z, \alpha)$$

We first establish a lemma.

LEMMA A2: *If $G_A(z) \succ_{ULR} G_B(z)$ then for all $\alpha \geq 0$, $P(z, \alpha)$ has two extremes, a minimum at $\hat{z}_-$ and a maximum at $\hat{z}_+$, such that $\underline{z} \leq \hat{z}_- < \tilde{z} < \hat{z}_+ < \bar{z}$.*

PROOF:

First, if $\alpha = 0$, then by Proposition 1 of Hopkins and Kornienko (2003), if $G_A \succ_{ULR} G_B$ then $P(z, 0)$ is unimodal with a unique maximum in $(\hat{z}, \bar{z}]$. That is, the above Lemma holds with the first extreme at the lower bound, that is, $\underline{z} = \hat{z}_-$. Then, note that whenever $P(z, 0) > 1$, $P(z, \alpha) > 1$ as well, and whenever $P(z, 0) < 1$, $P(z, \alpha) < 1$. Also note that $P(\underline{z}, \alpha) = P(\tilde{z}, \alpha) = P(\bar{z}, \alpha) = P(\tilde{z}, 0) = P(\bar{z}, 0) = 1$. Therefore, there is at least one extreme point for $P(z, \alpha)$ on each of the two intervals $(\underline{z}, \tilde{z})$ and $(\tilde{z}, \bar{z})$. Note that $\partial P(z, \alpha)/\partial z = 0$ if and only if $P(z, \alpha) = L(z)$. That is, $P(z, \alpha)$ and $L(z)$ cross at the turning points of $P(z, \alpha)$. Now since $L(z)$ is increasing on $(\underline{z}, \hat{z})$, at any crossing point $L(z)$ must cross $P(z, \alpha)$ from below and so there can only be one crossing point on $(\underline{z}, \hat{z})$. Equally there can be only one extreme for $P(z, \alpha)$ on $(\hat{z}, \underline{z})$ and the result follows.

Now, as noted above in footnote 24, we do not consider the nongeneric case of points of inflection in $L(z)$ or $P(z, \alpha)$ as they do not alter the qualitative relationship between $x_A(z)$ and $x_B(z)$. Let us now rule out any additional points of tangency between $x_A(z)$ and $x_B(z)$. Note that at any such point (if it exists), it must be that $g_A(z)/(G_A(z) + \alpha) = g_B(z)/(G_B(z) + \alpha)$. That is, we can have only two points of tangency, $\hat{z}_-$ and $\hat{z}_+$. Suppose this is the case, and we have two tangency points at the turning points of $P(\alpha, z)$. Without loss of generality, take $\hat{z}_-$ and suppose that $x_A(x) < x_B(x)$ in both the left and right ε-neighborhood of $\hat{z}_-$. Then, in the right ε-neighborhood of $\hat{z}_-$ we have that $x'_A(z) > x'_B(z)$, as $g_A(z)/(G_A(z) + \alpha) > g_B(z)/(G_B(z) + \alpha)$ and $\psi_A(z) > \psi_B(z)$. But this, together with $x_A(\hat{z}_-) = x_B(\hat{z}_-)$, implies that $x_A(z) > x_B(z)$ in the right ε-neighborhood of $\hat{z}_-$, and we have a contradiction. Thus, if $x_A(z)$ and $x_B(z)$ are equal at any point, it must be a point of crossing.

Let us now check if $x_A(z)$ and $x_B(z)$ cross at the turning points of $P(z, \alpha)$. Without loss of generality, let $x_A(\hat{z}_+) = x_B(\hat{z}_+)$. But this, together with $g_A(\hat{z}_+)/G_A(\hat{z}_+) = g_B(\hat{z}_+)/G_B(\hat{z}_+)$ implies that $x_A(z)$ and $x_B(z)$ are tangent at $\hat{z}_+$, which, as we have shown above, is impossible.

One can see that if $x_A(z)$ and $x_B(z)$ cross at all on the interior of $[\underline{z}, \bar{z}]$, then at such crossing points $\psi(x_A, z) = \psi(x_B, z)$. Whenever $g_A(z)/(G_A(z) + \alpha) < g_B(z)/(G_B(z) + \alpha)$ (or when $z \in [\underline{z}, \hat{z}_-)$ or $(\hat{z}_+, \bar{z}]$), we have that $x'_A(z) < x'_B(z)$, and whenever $g_A(z)/(G_A(z) + \alpha) > g_B(z)/(G_B(z) + \alpha)$ (when $z \in (\hat{z}_-, \hat{z}_+)$), we have that $x'_A(z) > x'_B(z)$ at the points of intersection. Therefore, it must be that $x_A(z)$ crosses $x_B(z)$ at most three times—from above to the left of $\hat{z}_-$ and to the right of $\hat{z}_+$, and from below in between. Denote these points of intersection z_1, z_2, and z_3 with $z_1 < \hat{z}_- < z_2 < \hat{z}_+ < z_3$. Then the sequence of sign of the difference $x_A - x_B$ is $+, -, +, -$.

Let us now show that there is no crossing point z_1 on $(\underline{z}, \hat{z}_-)$. By the boundary condition, $x_A(\underline{z}) = x_B(\underline{z})$. Then, if there is a crossing point z_1, there must exist a point $\check{z} \in (\underline{z}, z_1)$ where the difference $x_A(z) - x_B(z)$ is maximized. At this point, the slopes of $x_A(z)$ and $x_B(z)$ must be equal, i.e. $x'_A(\check{z}) = x'_B(\check{z})$. Since from (A5) we have $\partial\psi(x, z)/\partial x < 0$, then it must be that $\psi_A(\check{z}) < \psi_B(\check{z})$. But this implies that $g_A(z)/(G_A(z) + \alpha) > g_B(z)/(G_B(z) + \alpha)$ at $\check{z}$, which is a contradiction. Thus, $x_A(z)$ crosses $x_B(z)$ at most twice, first from below at z_2, to the right of $\hat{z}_-$, then from above at z_3, to the right of $\hat{z}_+$.

(i) If $\alpha = 0$, then we can also rule out a crossing at z_2. Suppose not, so that $x_B(z) > x_A(z)$ on the interval $(\underline{z}, z_2)$. As noted in the proof of Lemma A2, $P(z, 0)$ is strictly increasing on $(\underline{z}, \hat{z}_+)$, so that $g_A/G_A > g_B/G_B$ on $(\underline{z}, z_2]$ as $z_2 < \hat{z}_+$. Given that ψ is decreasing in x, together it would imply that $x'_A(z) > x'_B(z)$ on $(\underline{z}, z_2)$, which generates a contradiction. Therefore $x_A(z) > x_B(z)$ on the interval $(\underline{z}, \hat{z}_+)$. As we showed above, $x_A(\hat{z}_+) \neq x_B(\hat{z}_+)$, and thus, by continuity, $x_A > x_B$ on $(\underline{z}, \hat{z}_+]$.

(ii) Consider the case of $\alpha > 0$. Suppose $x_A(z) \geq x_B(z)$ for all z in $[\underline{z}, \hat{z}_-]$. Then, $\psi_A(z) \leq \psi_B(z)$ for all $[\underline{z}, \hat{z}_-]$. Together, with the fact that $g_A(z)/(G_A(z) + \alpha) < g_B(z)/(G_B(z) + \alpha)$ on $(\underline{z}, \hat{z}_-)$, we have $x'_A(z) < x'_B(z)$ on $(\underline{z}, \hat{z}_-)$, which contradicts the claim that $x_A(z) \geq x_B(z)$ for all z in $[\underline{z}, \hat{z}_-]$. Thus, instead it must be that $x_A(z) < x_B(z)$ for all z in $(\underline{z}, \hat{z}_-)$. Again, as $x_A(\hat{z}_-) \neq x_B(\hat{z}_-)$, by continuity we have that $x_A > x_B$ on $(\underline{z}, \hat{z}_-]$. We can show that x_A does cross over on the interval $(\hat{z}_-, \tilde{z})$. Suppose not. We would then have both $x_A(z) < x_B(z)$ and, as $G_A(z) \succ_{ULR} G_B(z)$, $G_A(z) < G_B(z)$ for all z in $(\underline{z}, \tilde{z})$, so that $U_A^{*\prime}(z) < U_B^{*\prime}(z)$ on that interval. Given $U_A^*(\underline{z}) = U_B^*(\underline{z})$, this would imply $U_A^*(\tilde{z}) < U_B^*(\tilde{z})$. But then as $G_A(\tilde{z}) = G_B(\tilde{z})$, for utility in society A to be lower at $\tilde{z}$, we must have $x_A(\tilde{z}) > x_B(\tilde{z})$, a contradiction. So, there is a crossing and in fact, $x_A(\tilde{z}) > x_B(\tilde{z})$.

PROOF OF PROPOSITION 5:

If $G_A \succ_{ULR} G_B$ then by Lemma A2, $g_A(z)/(G_A(z) + \alpha) > g_B(z)/(G_B(z) + \alpha)$ on $(\hat{z}_-, \hat{z}_+)$ and $g_A(z)/(G_A(z) + \alpha) < g_B(z)/(G_B(z) + \alpha)$ on $(\hat{z}, \bar{z}]$. Given the definition of $\tau(z)$ in (12), and that the cooperative solutions (x_c, y_c) are independent of the distribution of income, the result follows.

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